Identification-based first-order algorithms for distributed learning

Dmitry Grishchenko

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Collaborators

Collaborators

Collaborators

My supervisors

F. Iutzeler
LJK

J. Malick
CNRS, LJK

Collaborators

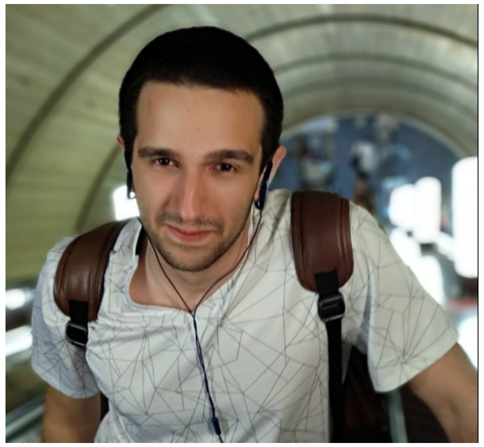
My supervisors

F. Iutzeler
LJK

J. Malick
CNRS, LJK

My student

L. Kochiev
MIPT



Problem

$$\min_{x \in \mathbb{R}^n} \sum_{i=1}^{M} f_i(x) + r(x)$$

Problem

$$\min_{x \in \mathbb{R}^n} \sum_{i=1}^{M} f_i(x) + r(x)$$

convex
L-smooth

convex
non-smooth

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In ML context

$$f_i(x) = \sum_{j \in S_i} \pi_i l_j(x)$$

l_j – loss associated with j^{th} example
S_i – i^{th} set of examples
π_i – proportion of examples in i^{th} set

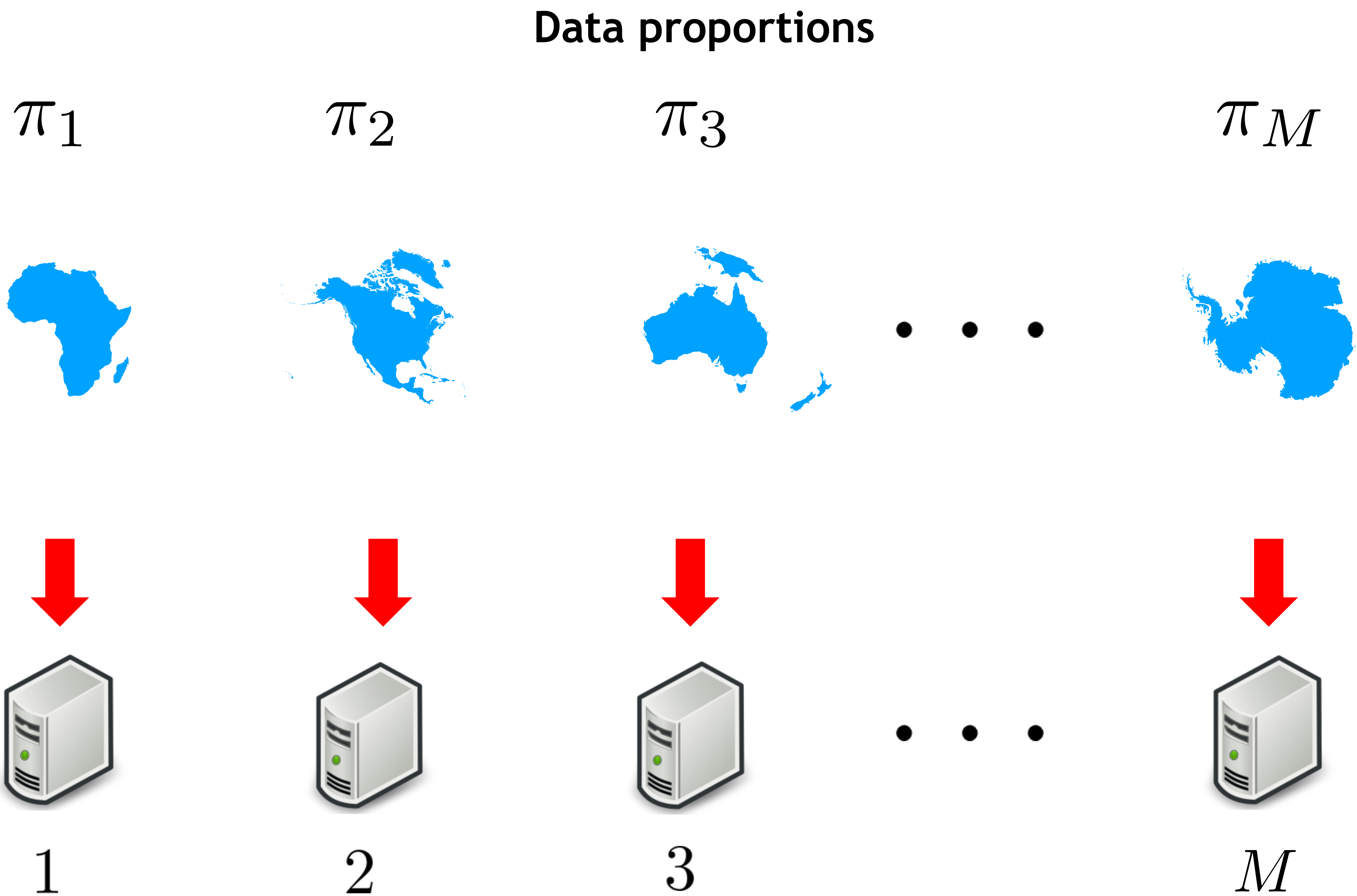
Distribute It!

Distribute It!

Data proportions

π_1 π_2 π_3 π_M

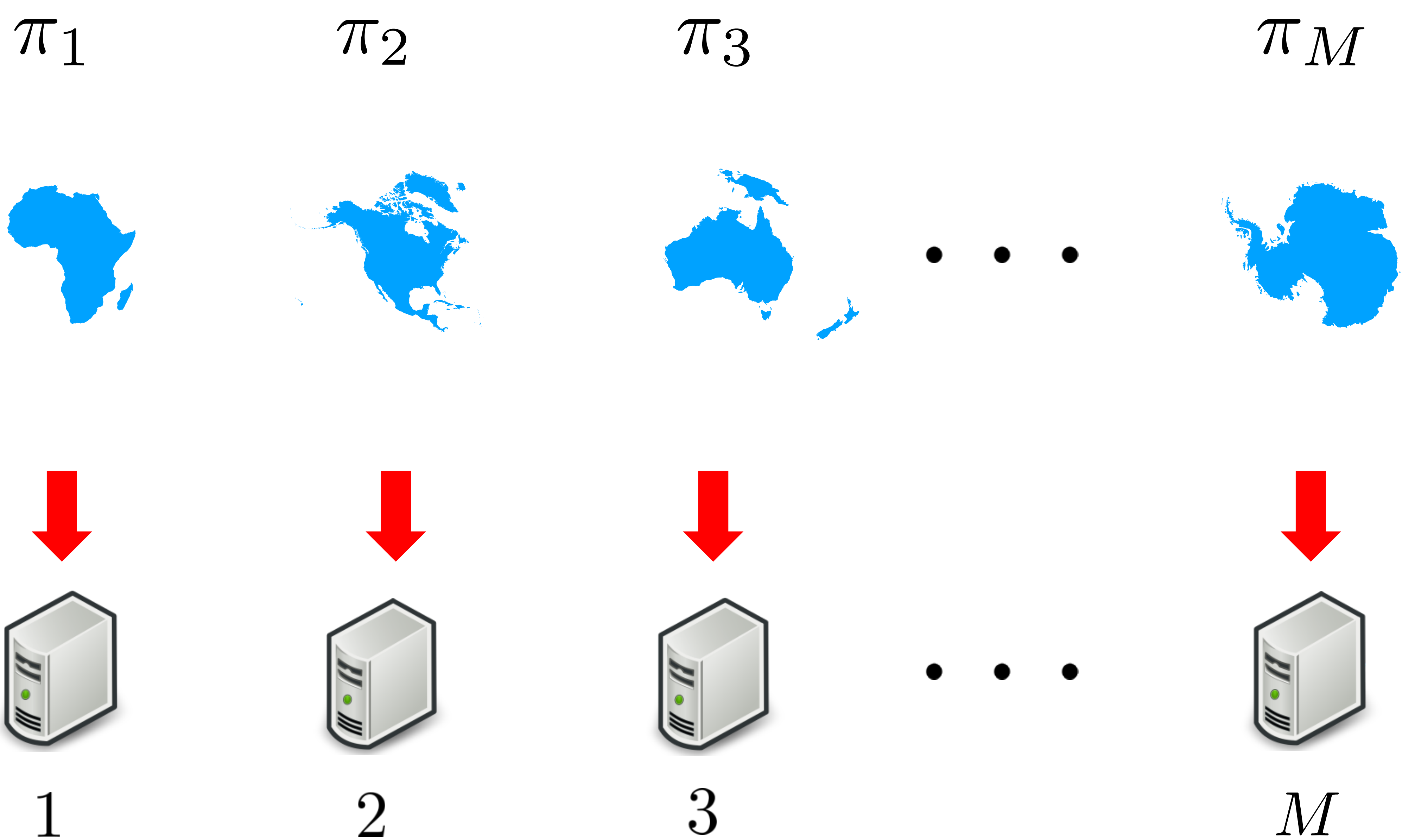
Distribute It!



Distribute It!

Centralized

Data proportions



Parallel or Distributed?

Parallel or Distributed?

	Parallel	Distributed

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	Parallel	Distributed
Machines		

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Machines	Single	Multiple

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Storage		

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Private data

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Machines = Phones (Federated Learning)

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Result

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Machines = Phones (Federated Learning)

Result <u>Computations are cheaper than communications</u>

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Result <u>Computations are cheaper than communications</u>

Let's make communications cheaper!

The Size Is Important!

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Not so many fans

The Size Is Important!

Not so many fans → Everyone can go to the stadium and watch the game

The Size Is Important!

Not so many fans → **Everyone can go to the stadium and watch the game**

The Size Is Important!

~~Not so many fans~~ → Everyone can go to the stadium and watch the game

A lot of fans

The Size Is Important!

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A lot of fans → Need to make a ballot for the tickets

The Size Is Important!

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~~Everyone can go to the stadium and watch the game~~

A lot of fans → Need to make a <u>ballot</u> for the tickets

<u>**sparsification**</u>

Synchronous?

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Drawbacks of synchronous algorithms

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Wasting of time

All machines have to wait the
moment, when everyone finishes

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Drawbacks of synchronous algorithms

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A lot of communications in one time

Master machine communicates with
all machines in the same time

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Let's kill 2 birds with one stone

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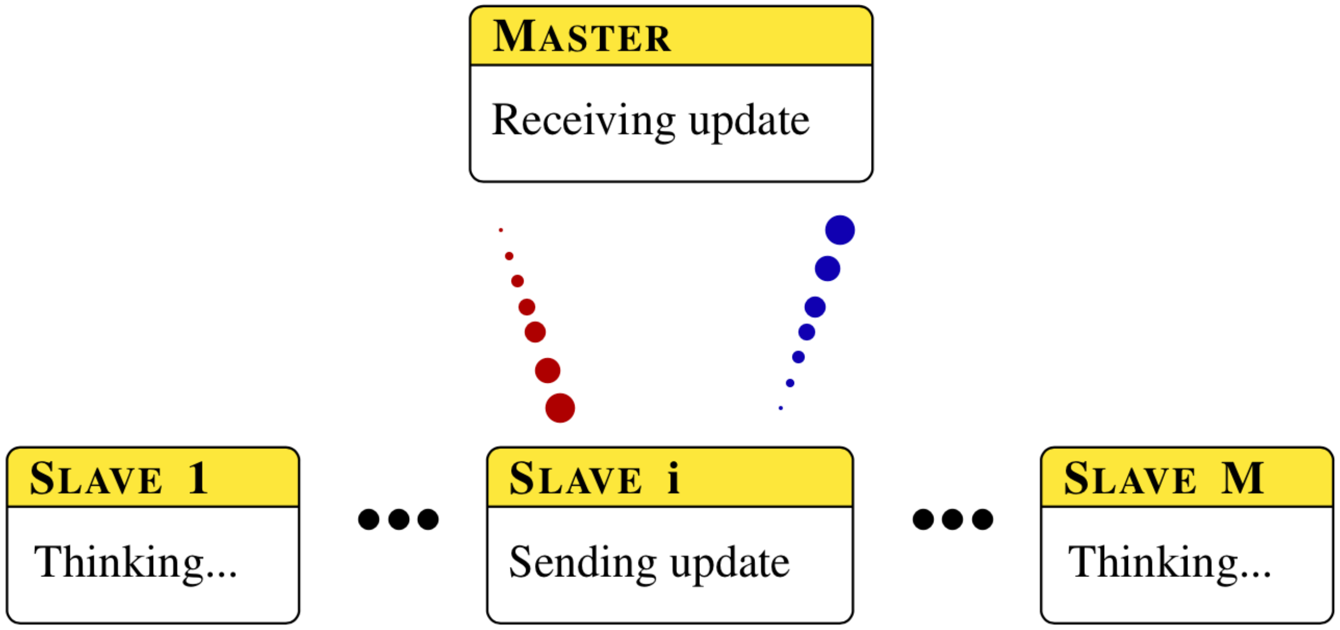

Back to Basics

Problems of the form

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First-order proximal methods (for example Proximal Gradient Descent)

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$$x^{k+1} = \underset{\gamma g}{\mathbf{prox}}(x^k - \gamma \nabla f(x^k))$$

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where $\mathbf{prox}_{\gamma g}(x) := \operatorname*{argmin}_u \left\{ g(u) + \frac{1}{2\gamma}\|u - x\|^2 \right\}$

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Each worker send its gradient and master calculate the weighted average

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$$F(x) = \sum_{i=1}^{M} \pi_i f_i(x) \longrightarrow \nabla F(x) = \sum_{i=1}^{M} \pi_i \nabla f_i(x)$$

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Asynchronous updates bring some delays

Gradient that master receive from worker is computed in one of previous iterate points

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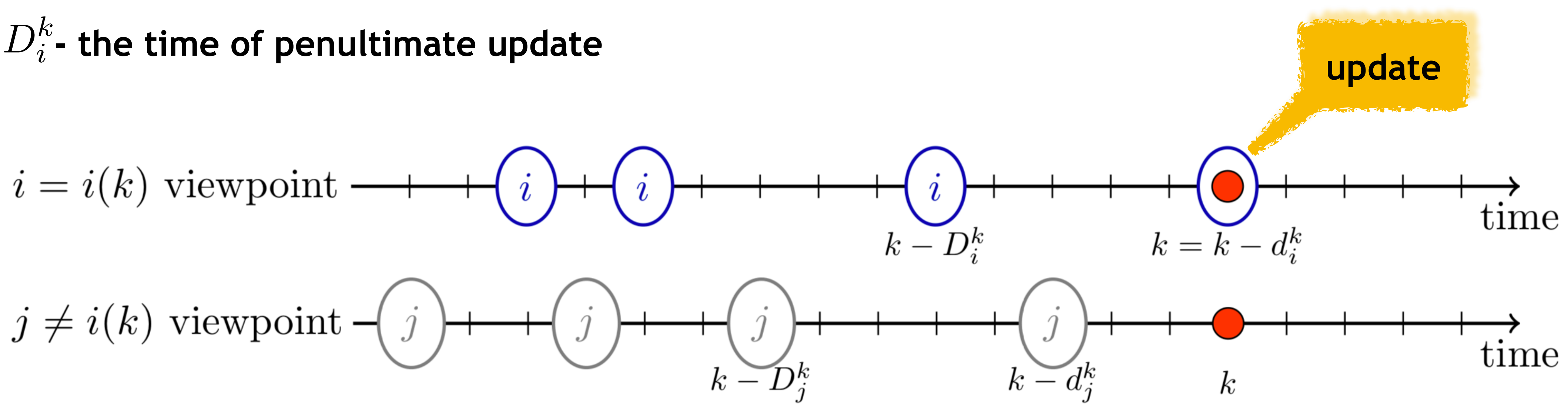
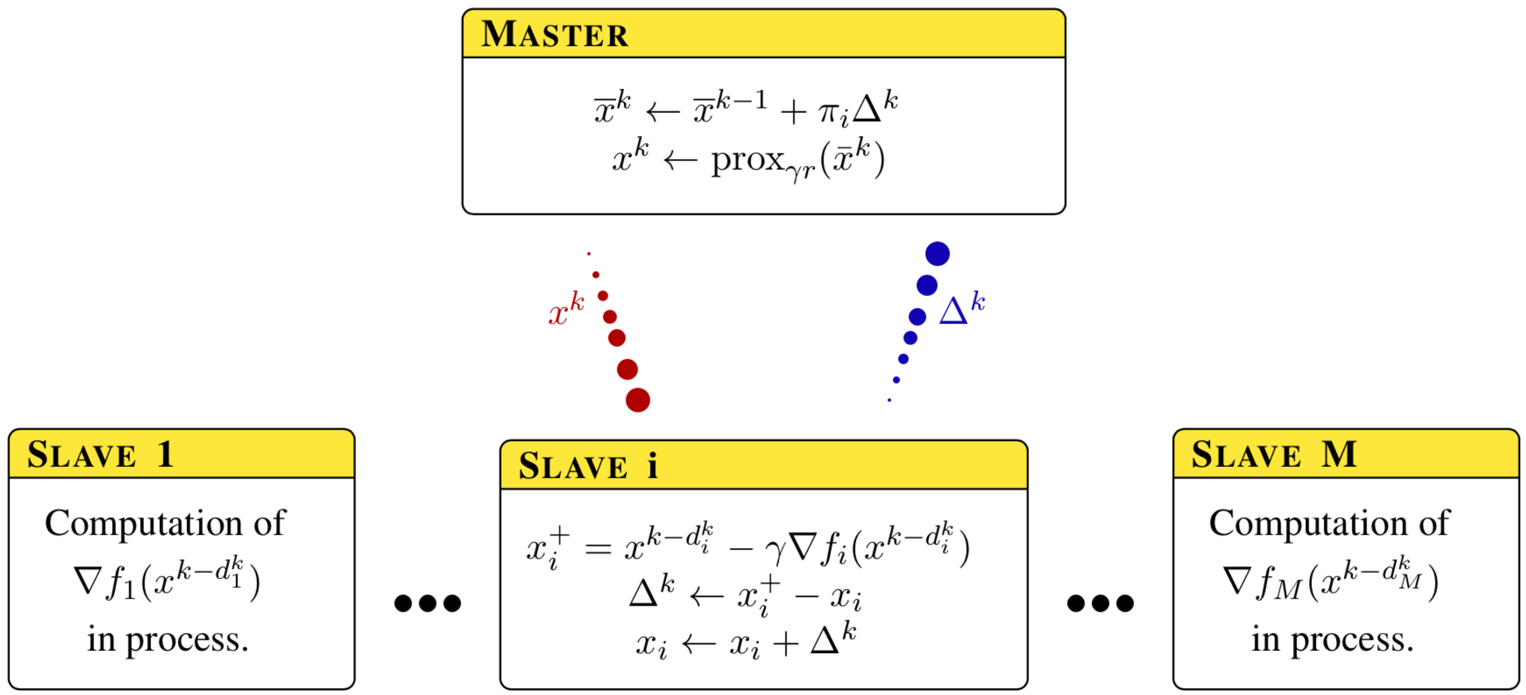


Non-sparsified Algorithm

Non-sparsified Algorithm



Mishchenko, K., Iutzeler, F., Malick, J., & Amini, M. R. (2018, July).
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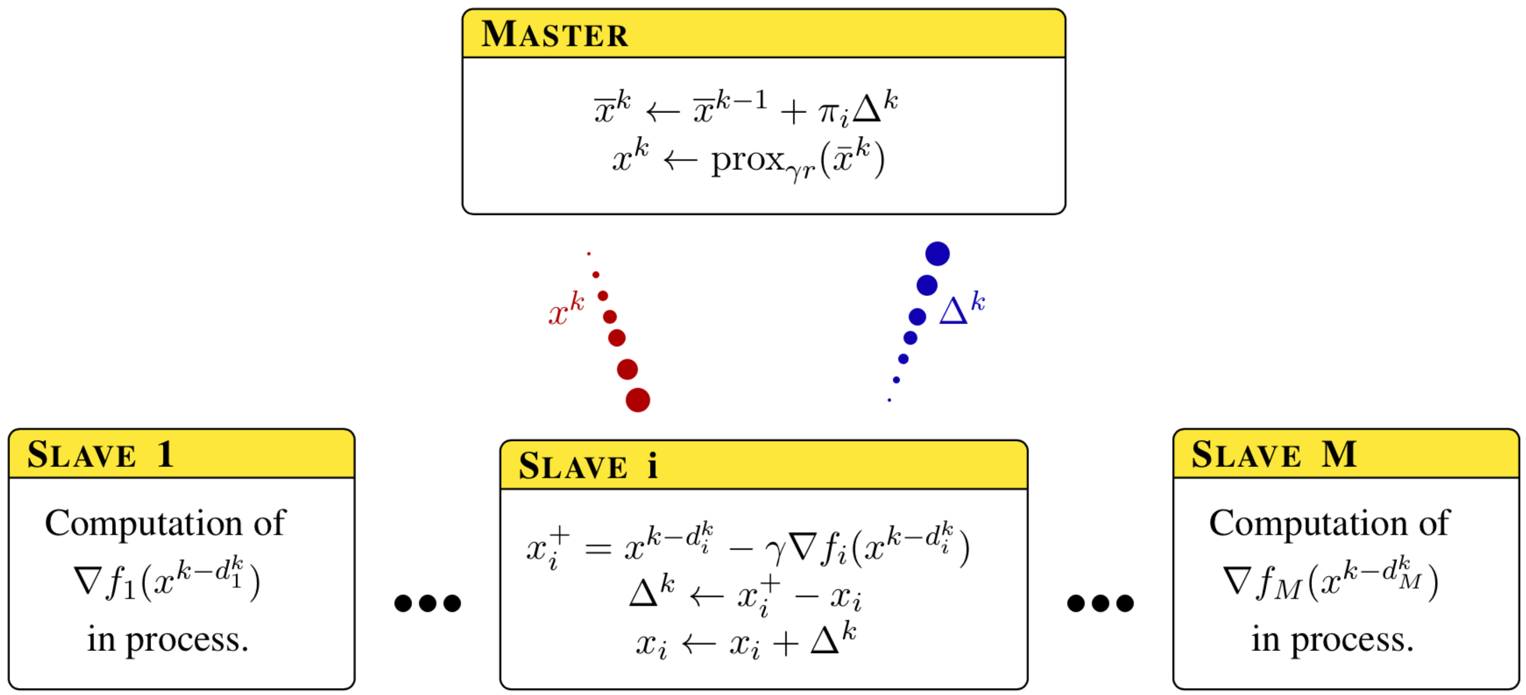

If dimension is very big, it is very expansive to send full gradient

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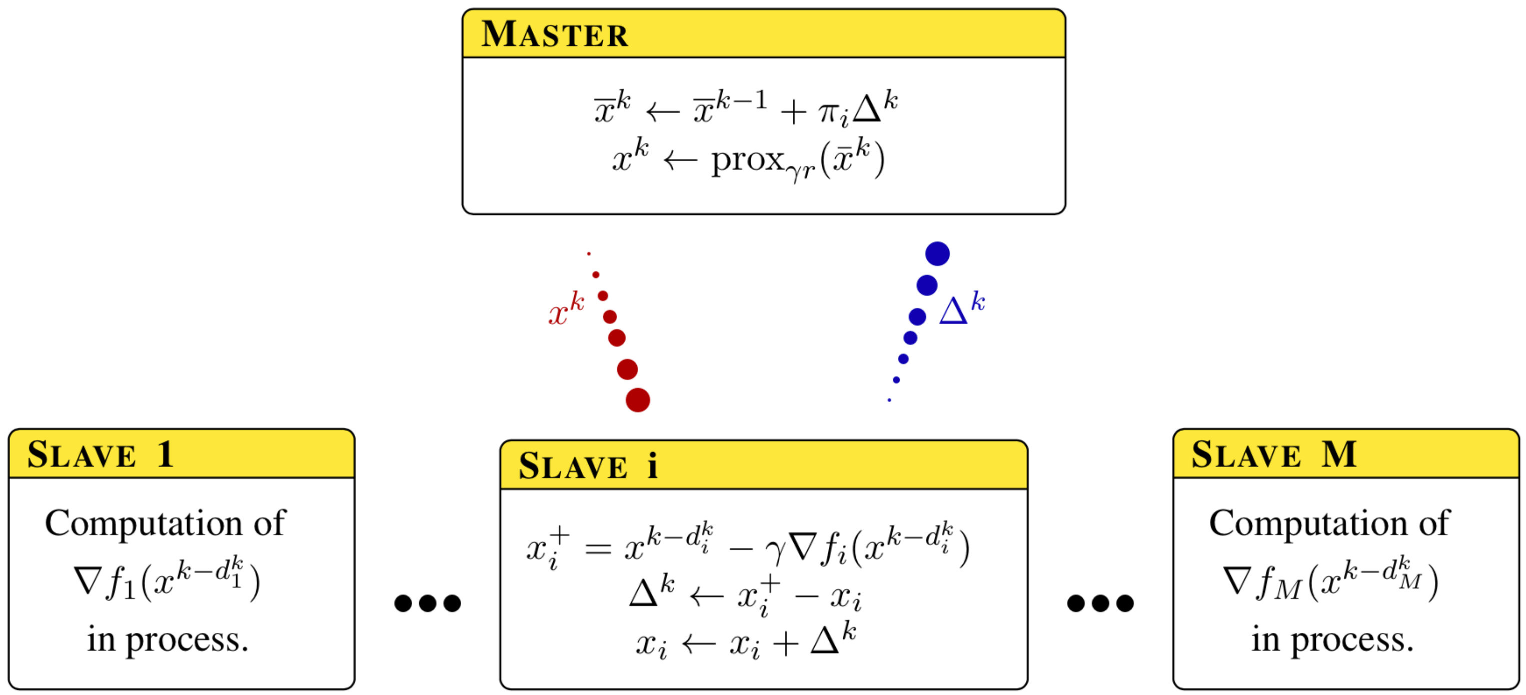

If dimension is very big, it is very expansive to send full gradient

If regulariser is sparsity enforcing, no need to sparsify master → worker

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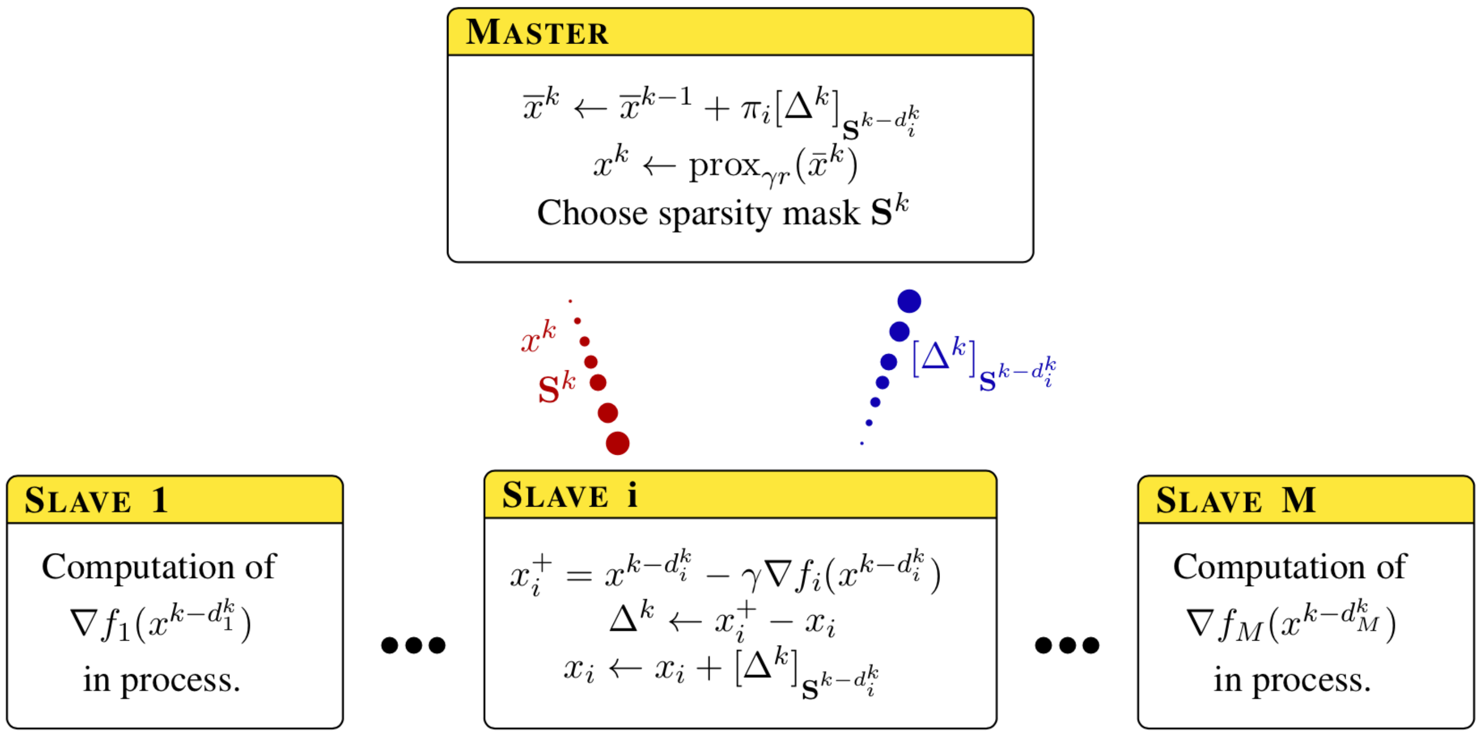
Fadili J., Malick J., Peyré G. Sensitivity analysis for mirror-stratifiable convex functions SIAM Journal on Optimization. – 2018. – T. 28. – №. 4. – C. 2975-3000.

Sparsified Algorithm

Sparsified Algorithm



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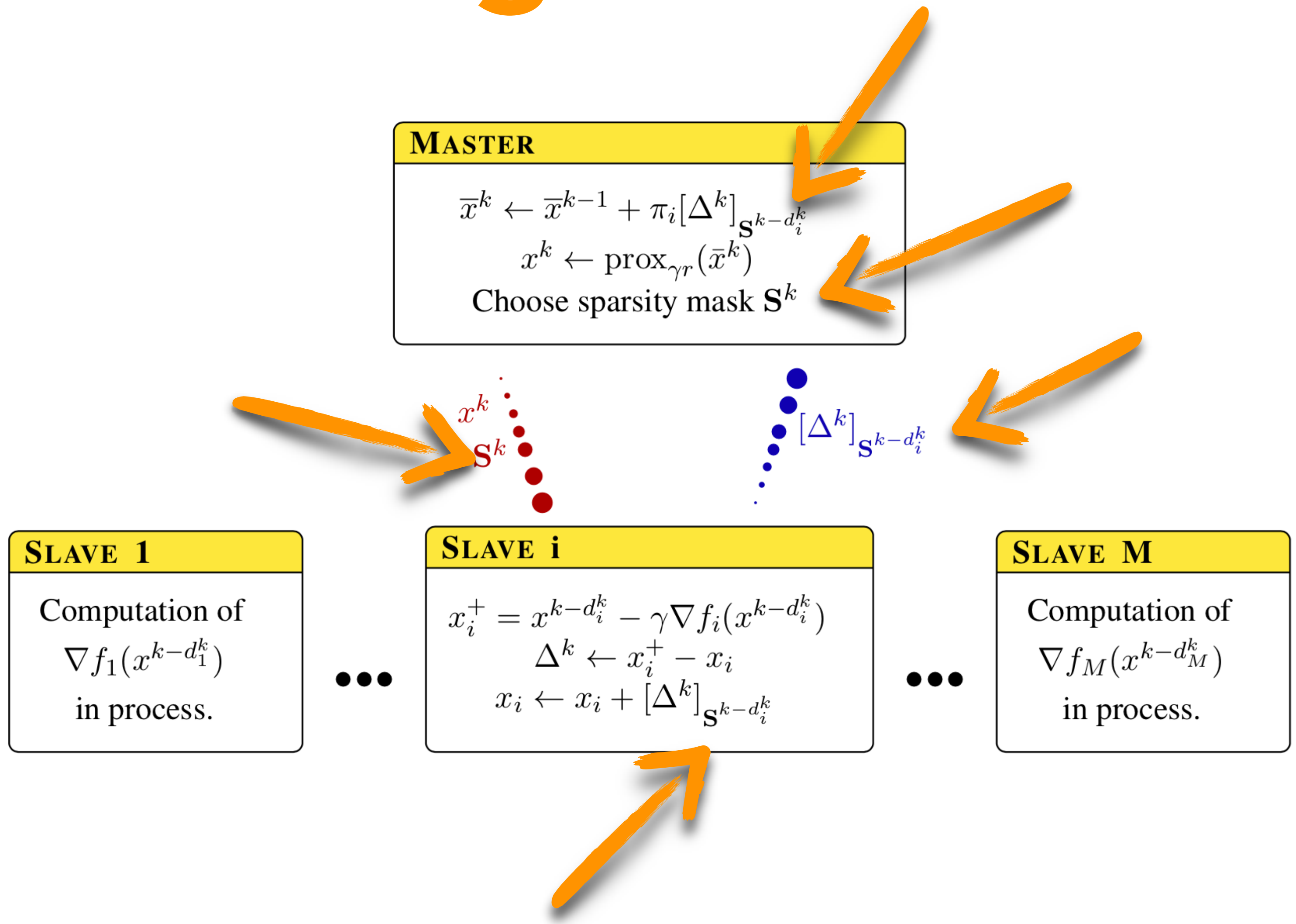

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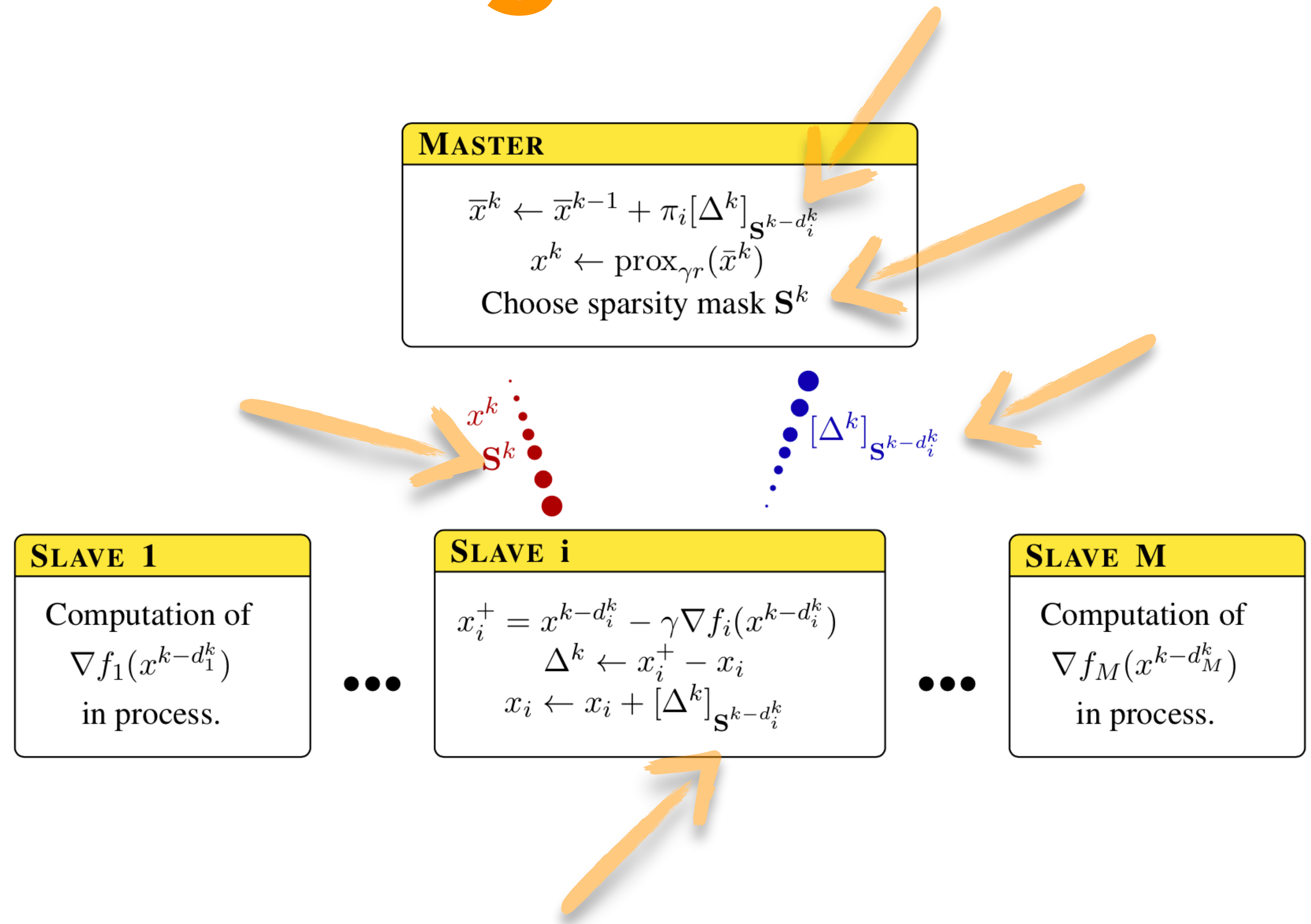

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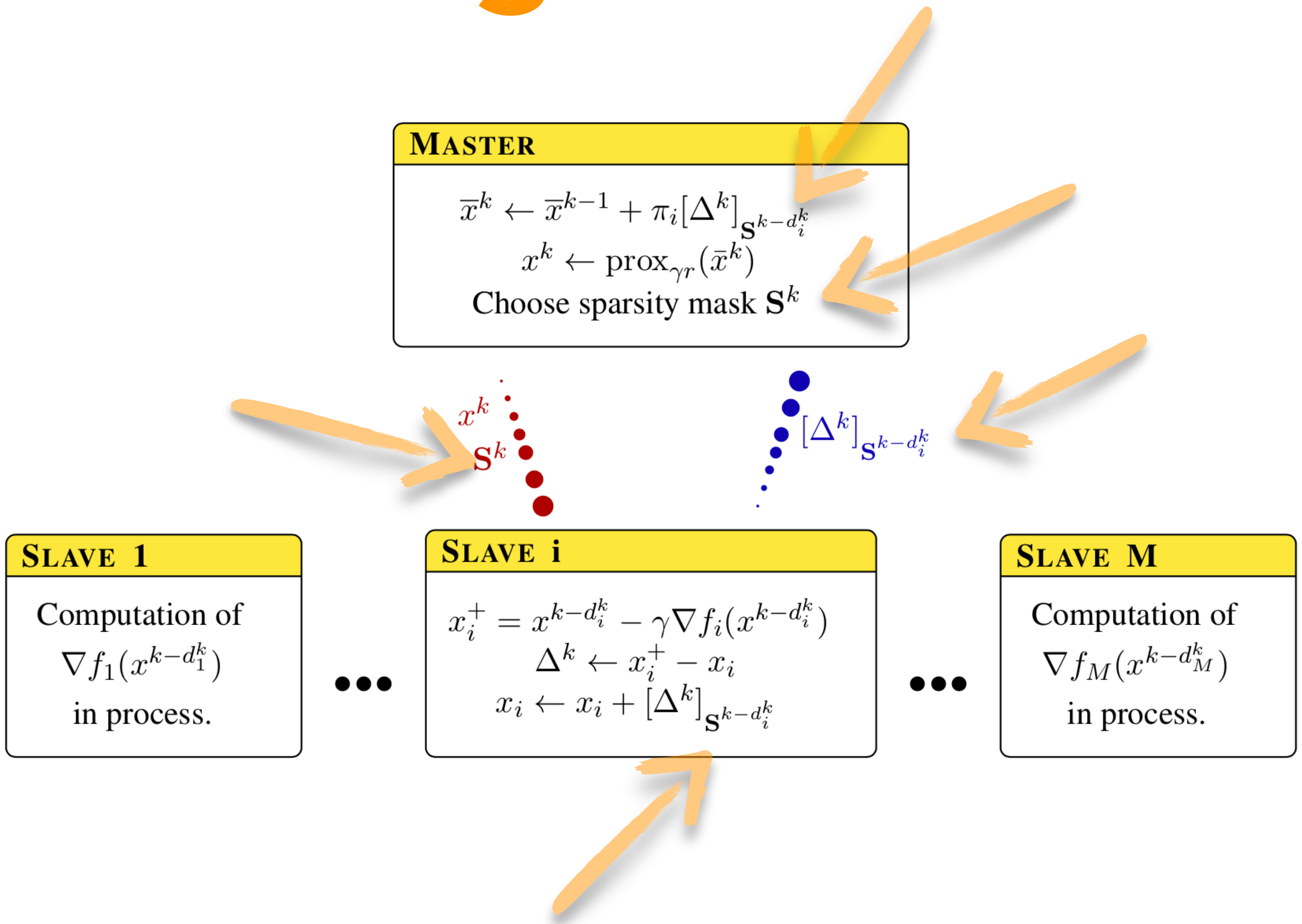
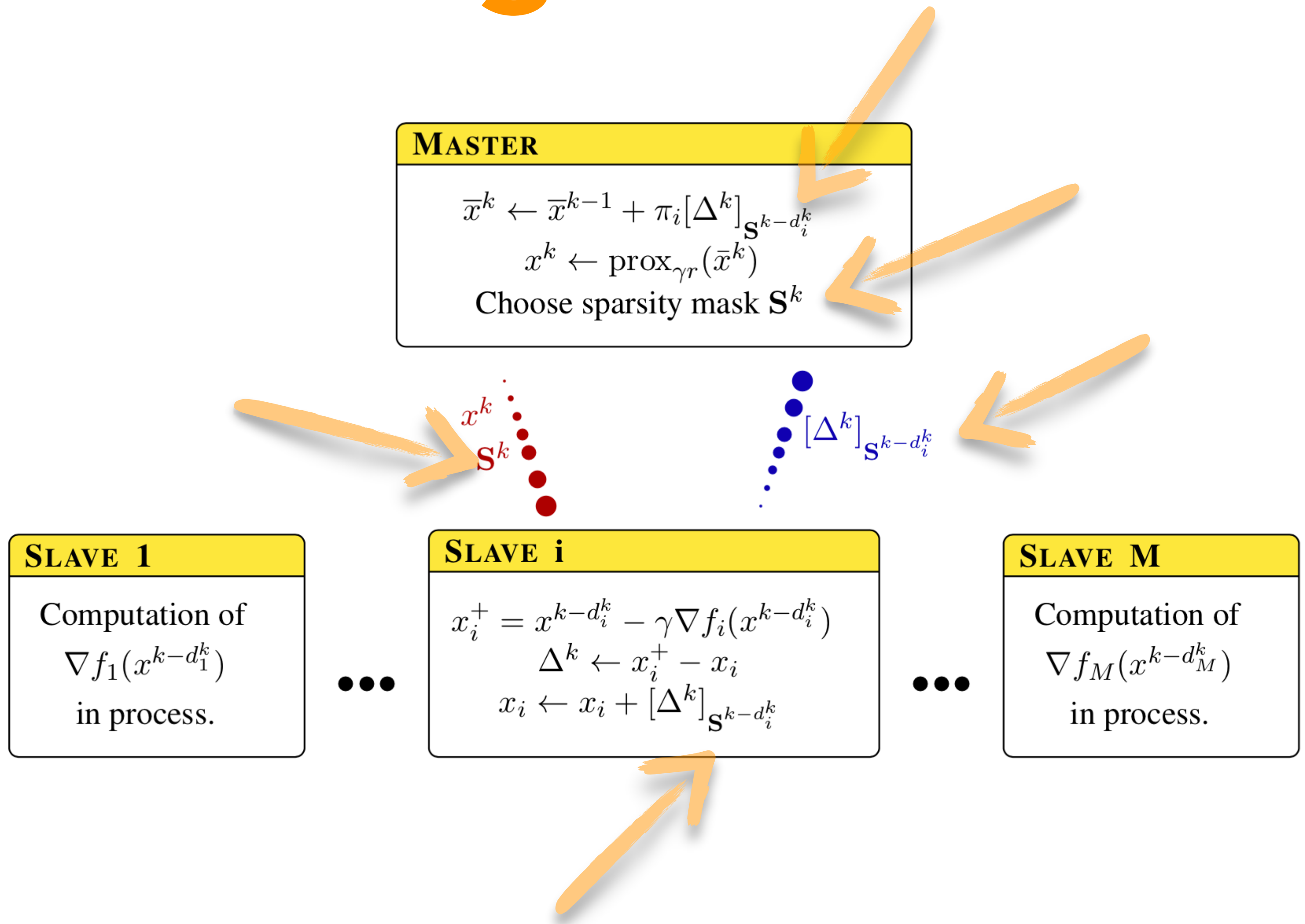


How to chose the sparsity mask?

Sparsified Algorithm



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Random uniform coordinate selection (Coordinate Descent with uniform probabilities)

Nesterov Y. Efficiency of coordinate descent methods on huge-scale optimization problems
SIAM Journal on Optimization. – 2012. – Т. 22. – №. 2. – С. 341-362.

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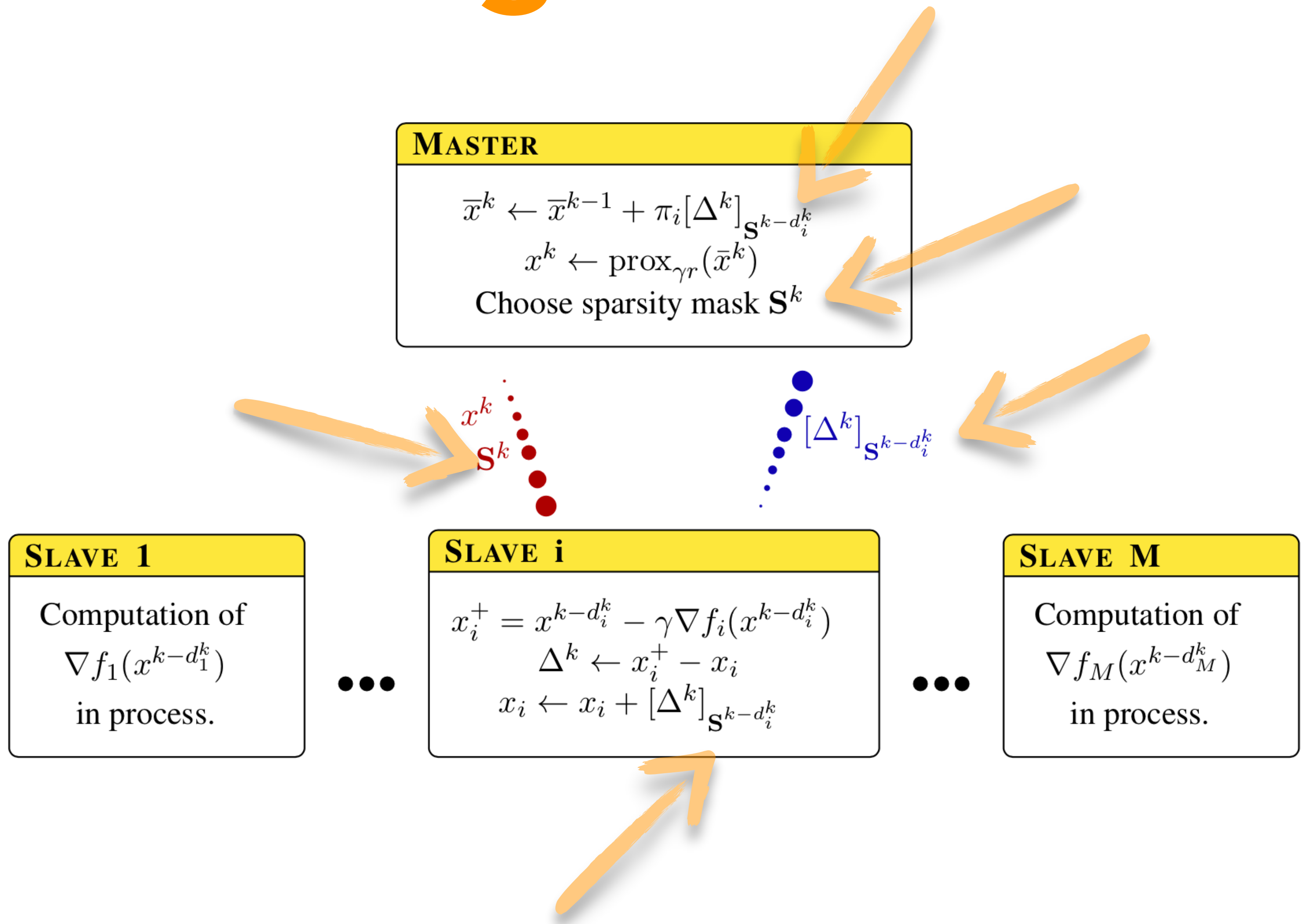

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OR…

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Back to Ballot

It’s an example of identification from real world

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1 wave - most loyal fans (the biggest amount of games visited)

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2 wave (after cancellations) - uniformly random from all

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This system gives a chance for everyone to watch a game

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This system gives a chance for everyone to watch a game

Most loyal have probability 1, all the others much smaller

It’s an example of identification from real world

Core Idea

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* - l_1-regularizer enforces coordinate sparsity

Core Idea

Let's introduce our adaptive way of mask selection for l_1-regularizer

,

.

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1 - select all the coordinates that are in master's current iterate

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1 - select all the coordinates that are in master's current iterate

2 - add some other coordinates with some probabilities

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As a result, all the coordinates from the support of the final solution would be selected

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Dimension reductions without loss of speed (but only from some finite moment of time)

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Dimension reductions without loss of speed (but only from some finite moment of time)

Theoretical result (for s.c. objective)

$$\mathbb{E}\|x_i^k - x_i^\star\|^2 \le \left(2 - 2\frac{\gamma\mu L}{\mu + L} - \min_i p_i\right) \max_{j=1,..,M} \mathbb{E}\left\|x_j^{k-D_i^k} - x_j^\star\right\|^2$$

where $k \in [k_m, k_{m+1})$, $k_{m+1} = \min\left\{k : k - D_i^k \ge k_m \text{ for all } i\right\}$, **and stepsize** $\gamma \in (0, 2/(\mu + L)]$.

Grishchenko, D., Iutzeler, F., Malick, J., & Amini, M. R. (2018).
Asynchronous Distributed Learning with Sparse Communications and Identification.
arXiv preprint arXiv:1812.03871.

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Core Idea v.2

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should be less than 1!

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Bound on the minimal probability that depends on problem conditioning

$$\min_i p_i \ge 1 - \frac{2\gamma\mu L}{\mu L}$$

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For ill-conditioned problems there is NO sparsification

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For ill-conditioned problems there is NO sparsification

Let us use l_2-regularizer to recondition the initial problem

Catalyst

Catalyst

Input: $\boldsymbol{x_0} \in \mathbb{R}^n$, smoothing parameter $\boldsymbol{\kappa}$, optimization method $\mathcal{M}$, $\boldsymbol{y_0} = \boldsymbol{x_0}$, $\boldsymbol{q} = \frac{\mu}{\mu + \boldsymbol{\kappa}}$

Output: $\boldsymbol{x}^\star \in \operatorname{argmin}_{x \in \mathbb{R}^n} f(x)$

while *desired stopping criterion is not satisfied* **do**

Find $\boldsymbol{x_k}$ using $\mathcal{M}$

$$\boldsymbol{x_k} \in \underset{x \in \mathbb{R}^n}{\operatorname{argmin}_{\boldsymbol{\varepsilon^k}}} \{ h_{\boldsymbol{k}}(x) \triangleq f(x) + \frac{\boldsymbol{\kappa}}{2} \| x - \boldsymbol{y_{k-1}} \|^2 \}$$

Compute $\boldsymbol{\alpha_k} \in (0;1)$ from $\boldsymbol{\alpha_k}^2 = (1 - \boldsymbol{\alpha_k}) \boldsymbol{\alpha_{k-1}}^2 + q \boldsymbol{\alpha_k}$

Compute $\boldsymbol{y_k}$ using $\boldsymbol{\beta_k}$ from $(0, 1)$

$$\boldsymbol{y_k} = \boldsymbol{x_k} + \boldsymbol{\beta_k} (\boldsymbol{x_k} - \boldsymbol{x_{k-1}}),$$

where

$$\boldsymbol{\beta_k} = \frac{\boldsymbol{\alpha_{k-1}} (1 - \boldsymbol{\alpha_{k-1}})}{\boldsymbol{\alpha_{k-1}}^2 + \boldsymbol{\alpha_k}}$$

end

Lin, H., Mairal, J., & Harchaoui, Z. (2015).A universal catalyst for first-order optimization. In *Advances in neural information processing systems* (pp. 3384-3392).

Catalyst

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Compute $\boldsymbol{\alpha_k} \in (0;1)$ from $\boldsymbol{\alpha_k}^2 = (1 - \boldsymbol{\alpha_k})\boldsymbol{\alpha_{k-1}}^2 + q\boldsymbol{\alpha_k}$
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end

The only requirement: method $\mathcal{M}$ that solves $\min\limits_x h_k(x)$ with <u>linear</u> rate.

Lin, H., Mairal, J., & Harchaoui, Z. (2015).A universal catalyst for first-order optimization. In *Advances in neural information processing systems* (pp. 3384-3392).

Inner Method

Inner Method

WORKER i

Initialize $\boldsymbol{\kappa}$
Receive x, y from master
Initialize $x_i = x_i^+ = x$
Update objective function

$$h_i(\cdot) = f_i(\cdot) + \frac{\boldsymbol{\kappa}}{2}\|\cdot - y\|^2$$

while *not interrupted by master* **do**
- $[x_i^+]_{\mathbf{S}} \leftarrow [x - \gamma \nabla h_i(x)]_{\mathbf{S}}$
- $\Delta \leftarrow x_i^+ - x_i$
- Send $[\Delta]_{\mathbf{S}}$ to master
- $[x_i]_{\mathbf{S}} \leftarrow [x_i^+]_{\mathbf{S}}$
- Receive x and $\mathbf{S}$ from master

end

Inner Method

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Initialize $\boldsymbol{\kappa}$

Receive x, y from master

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- Send $[\Delta]_{\mathbf{S}}$ to master
- $[x_i]_{\mathbf{S}} \leftarrow [x_i^+]_{\mathbf{S}}$
- Receive x and $\mathbf{S}$ from master

end

Inner Method

MASTER

Reinitialize $k = 1, \bar{x}_l^0 = \boldsymbol{y_{l-1}}$
Broadcast $x_l^0 = \mathbf{prox}_{\gamma g}\left(\bar{x}_l^0\right), \boldsymbol{y_{l-1}}$
while *stopping criterion is not satisfied* **do**
  Receive $[\Delta^k]_{\mathbf{S}^{k-D^k_{i^k}}}$ from agent i^k
  $\bar{x}_l^k \leftarrow \bar{x}_l^{k-1} + \pi_i [\Delta^k]_{\mathbf{S}^{k-D^k_{i^k}}}$
  $x_l^k \leftarrow \mathbf{prox}_{\gamma r}(\bar{x}_l^k)$
  Draw sparsity mask $\mathbf{S}^k$
  Send $x_l^k, \mathbf{S}^k$ to agent i^k
  $k \leftarrow k+1$
end
while *some workers compute* **do**
  Receive $[\Delta^k]_{\mathbf{S}^{k-D^k_{i^k}}}$ from agent i^k
  $\bar{x}_l^k \leftarrow \bar{x}_l^{k-1} + \pi_i [\Delta^k]_{\mathbf{S}^{k-D^k_{i^k}}}$
  Stop worker
end
$\boldsymbol{x_l} \leftarrow \mathbf{prox}_{\gamma g}(\bar{x}_l^k)$

WORKER i

Initialize $\boldsymbol{\kappa}$
Receive x, y from master
Initialize $x_i = x_i^+ = x$
Update objective function

$$h_i(\cdot) = f_i(\cdot) + \frac{\boldsymbol{\kappa}}{2}\| \cdot - y\|^2$$

while *not interrupted by master* **do**
  $[x_i^+]_{\mathbf{S}} \leftarrow [x - \gamma \nabla h_i(x)]_{\mathbf{S}}$
  $\Delta \leftarrow x_i^+ - x_i$
  Send $[\Delta]_{\mathbf{S}}$ to master
  $[x_i]_{\mathbf{S}} \leftarrow [x_i^+]_{\mathbf{S}}$
  Receive x and $\mathbf{S}$ from master
end

Fly in the Ointment

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Required size of update*

*** - in the beginning (when the support is big enough) coordinates from the support could be also selected with some probability**

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Bound on $\boldsymbol{\kappa} \geq \dfrac{(1 - \min_i p_i)L - \mu}{\min_i p_i}$

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Requires more restarts than Catalyst with optimal parameter $\boldsymbol{\kappa}^\star$

if minimal probability is small enough and it's fair to tell about sparsification

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Communication Metric

after identification happened

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ρ **- the sparsity of the optimal solution**

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$|\boldsymbol{S}^k| \geq \rho n(1 + (1-\rho)\min_i p_i)$ **- … from worker**

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Results:

No reason to select coordinates with different probabilities

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Results:

No reason to select coordinates with different probabilities

An optimal uniform probability is the following

$$p^\star = \frac{2\rho}{1+3\rho}$$

Theoretical Result

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Theorem (s.c. case)

Consider the sparsity of the optimal solution ρ.

Choose $p = \dfrac{2\rho}{1+3\rho}$**, and corresponding** $\boldsymbol{\kappa} = \dfrac{(1-p)L-\mu}{p}$**.**

Then for any $\gamma \in \left(0, \dfrac{2}{\mu + L + 2\kappa}\right]$ **ASPY-DR algorithm**

converges $\tilde{O}\left(\sqrt{\dfrac{1+\rho}{\rho}}\right)$ **times faster in communications**

metrics than nonsparsified one.

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Amount of workers: 4

Problem: standard regularized logistic regression

Dataset: Madelon

Final sparsity: 0.05

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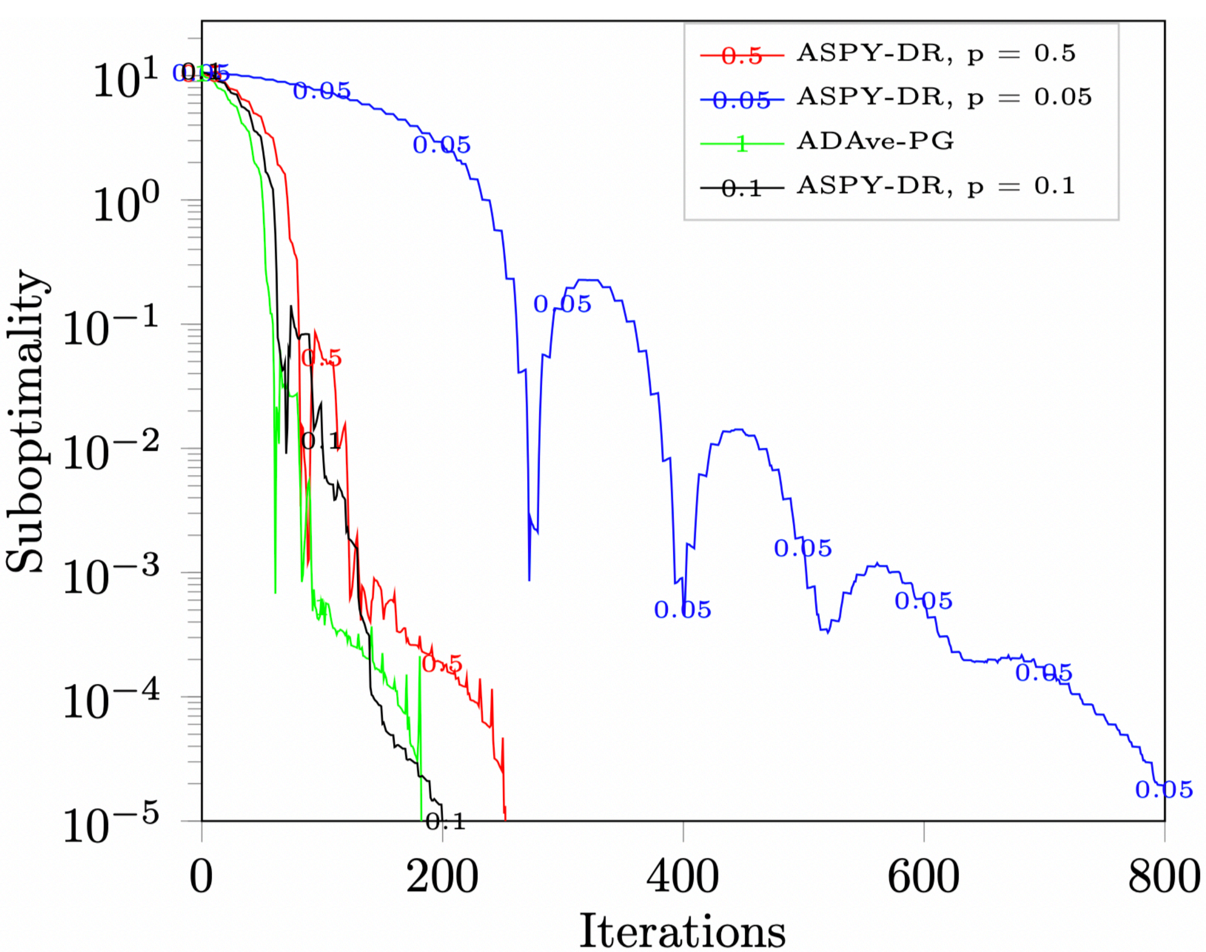

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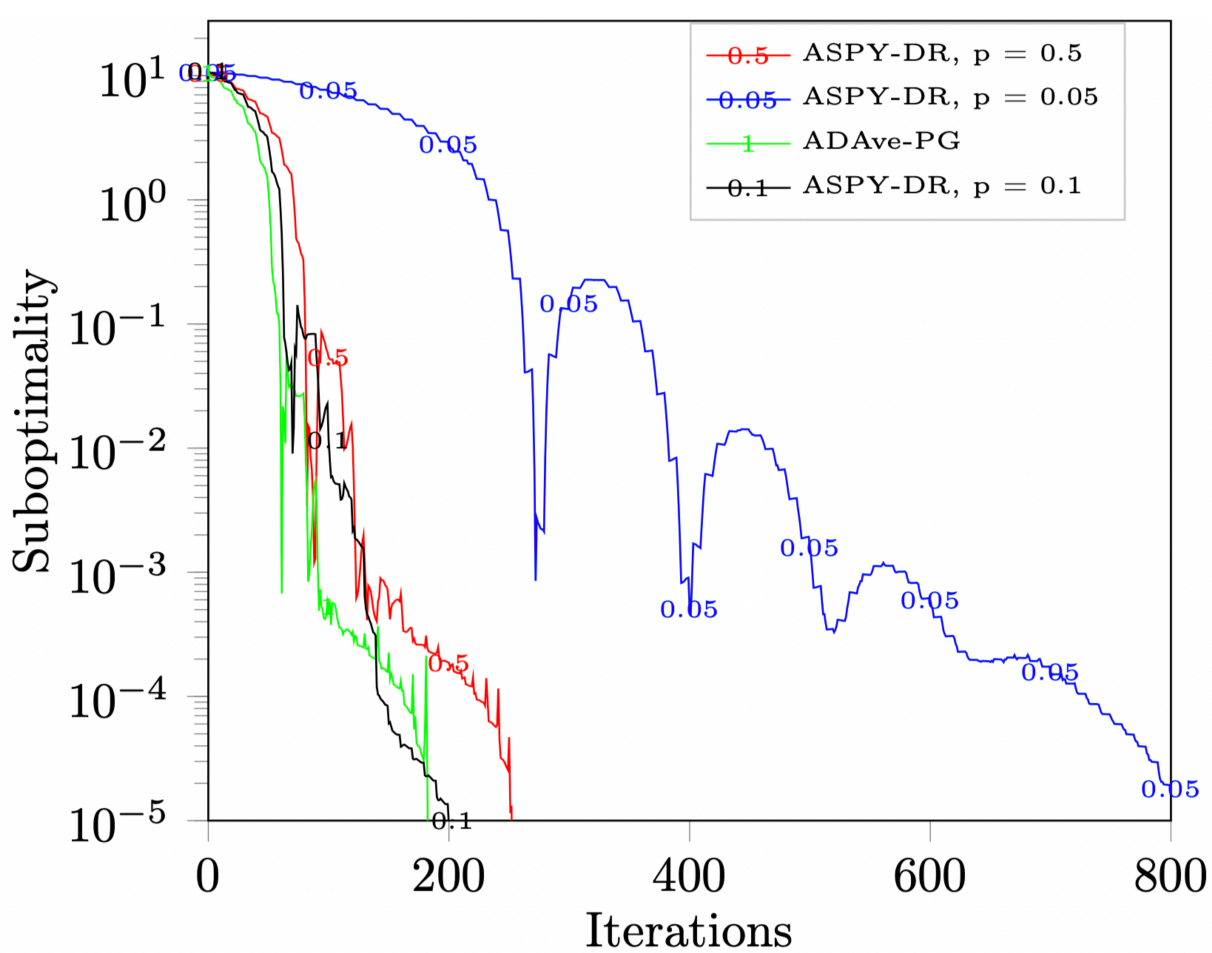

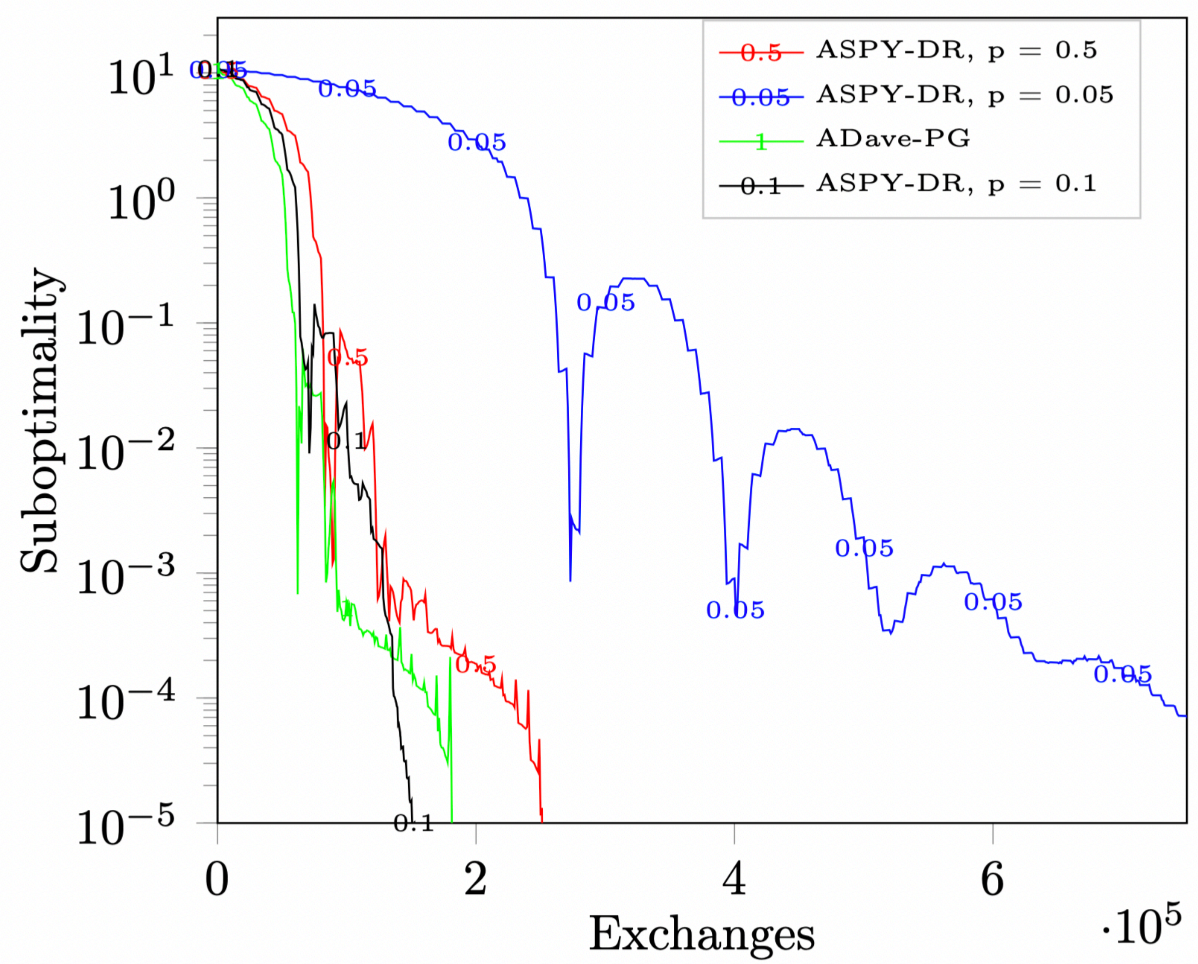

Thank You For Your Attention