Identification-based first-order algorithms for distributed learning

Dmitry Grishchenko

Problem

$$\min_{x \in \mathbb{R}^n} \sum_{i=1}^{M} f_i(x) + r(x)$$

Problem

convex
L-smooth

$$\min_{x \in \mathbb{R}^n} \sum_{i=1}^{M} f_i(x) + r(x)$$

convex
non-smooth

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In ML context

$$f_i(x) = \sum_{j \in S_i} \pi_i l_j(x)$$

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In ML context

$$f_i(x) = \sum_{j \in S_i} \pi_i l_j(x)$$

l_j – loss associated with j^{th} example
S_i – i^{th} set of examples
π_i – proportion of examples in i^{th} set

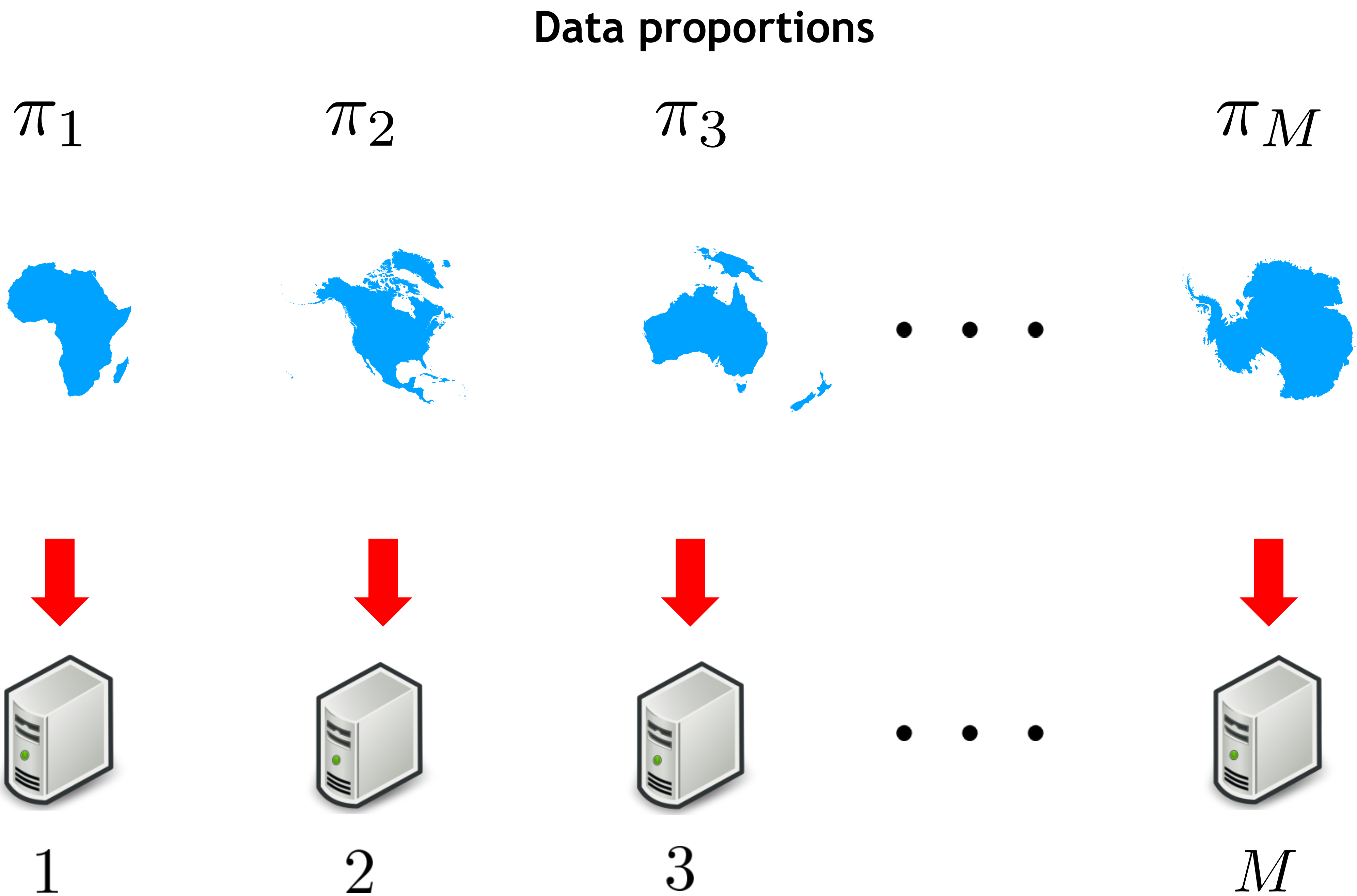
Distribute It!

Distribute It!

Data proportions

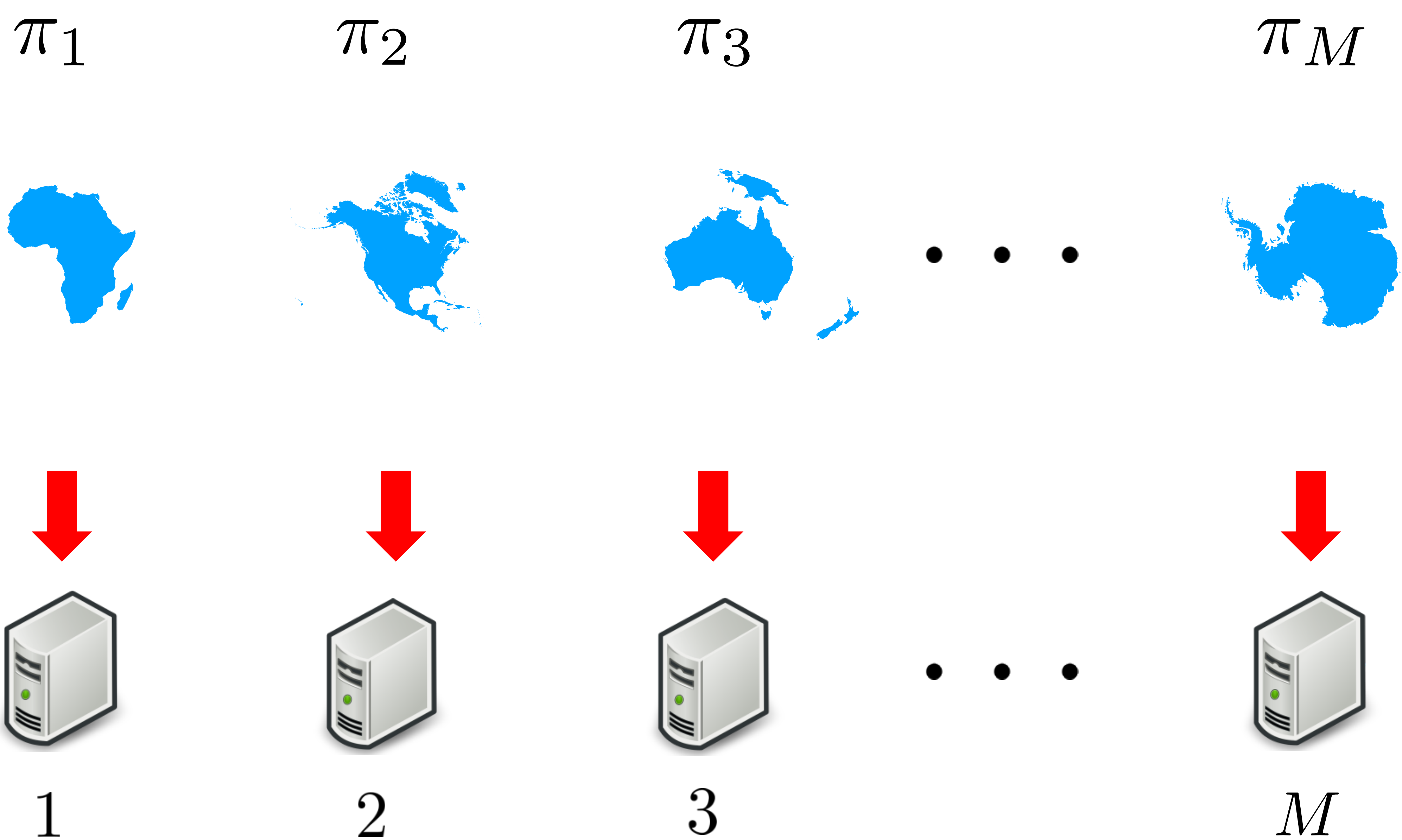
π_1 π_2 π_3 π_M

Distribute It!



Distribute It! Centralized

Data proportions



Parallel or Distributed?

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What is a difference?

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	Parallel	Distributed

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What is a difference?

	Parallel	Distributed
Machines		

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What is a difference?

	Parallel	Distributed
Machines	Single	Multiple

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Storage		

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Communication		

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Communication	Free	Bottleneck

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Private data

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Phones = Machines

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Result

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Multiple

Private data

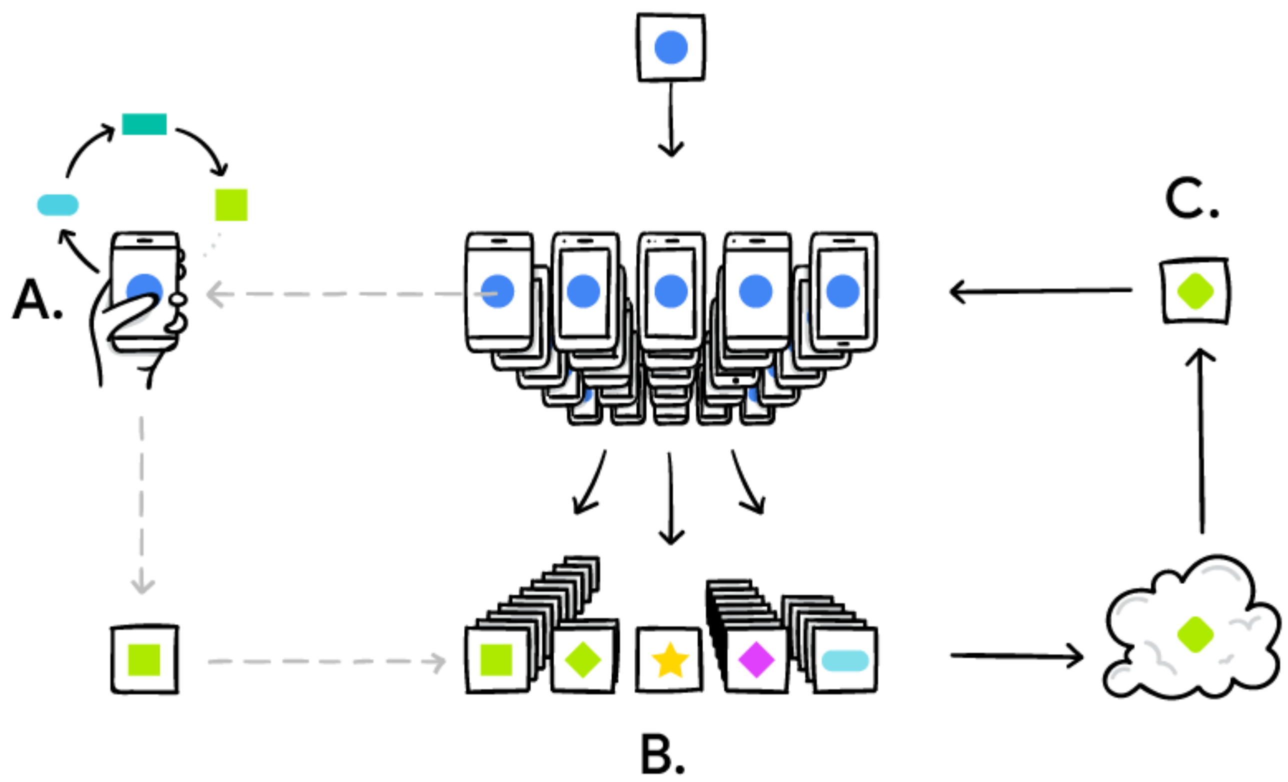
Phones = Machines

Result

<u>Computations are cheaper than communications</u>

Federated Learning

Federated Learning



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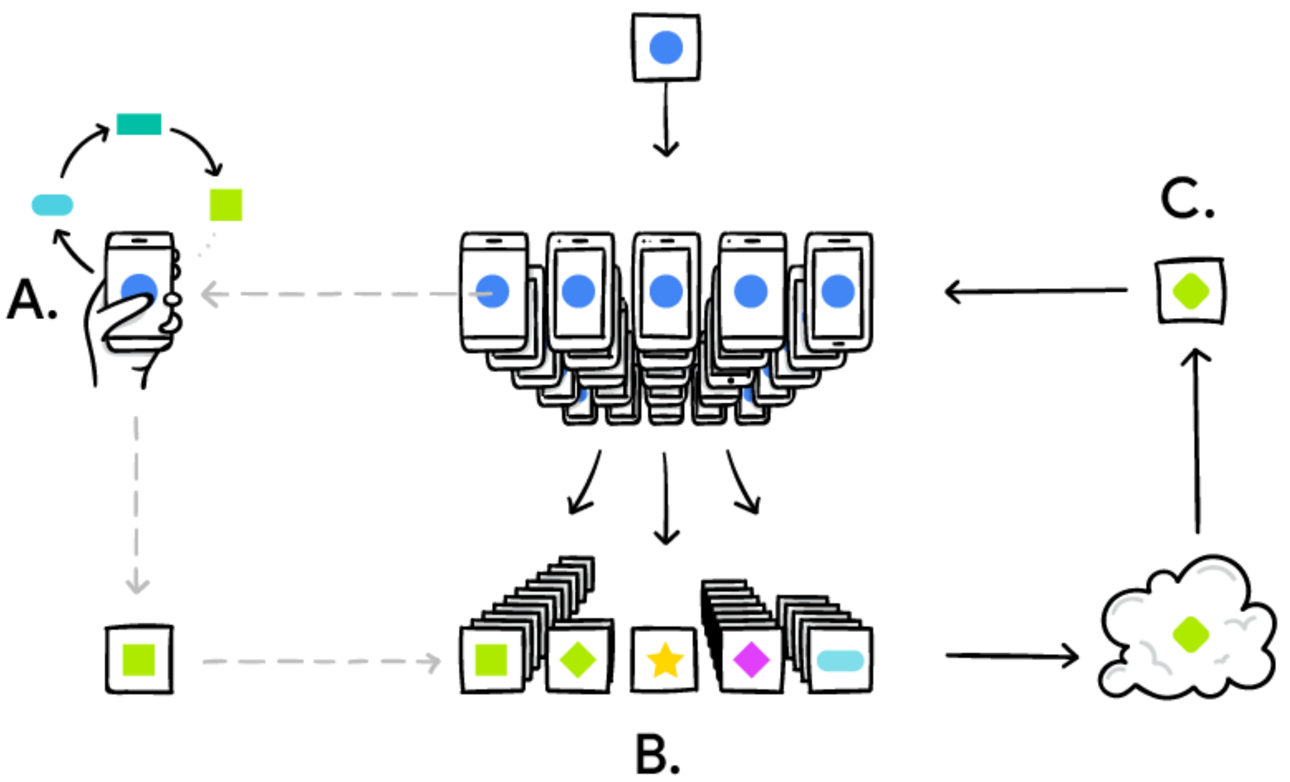
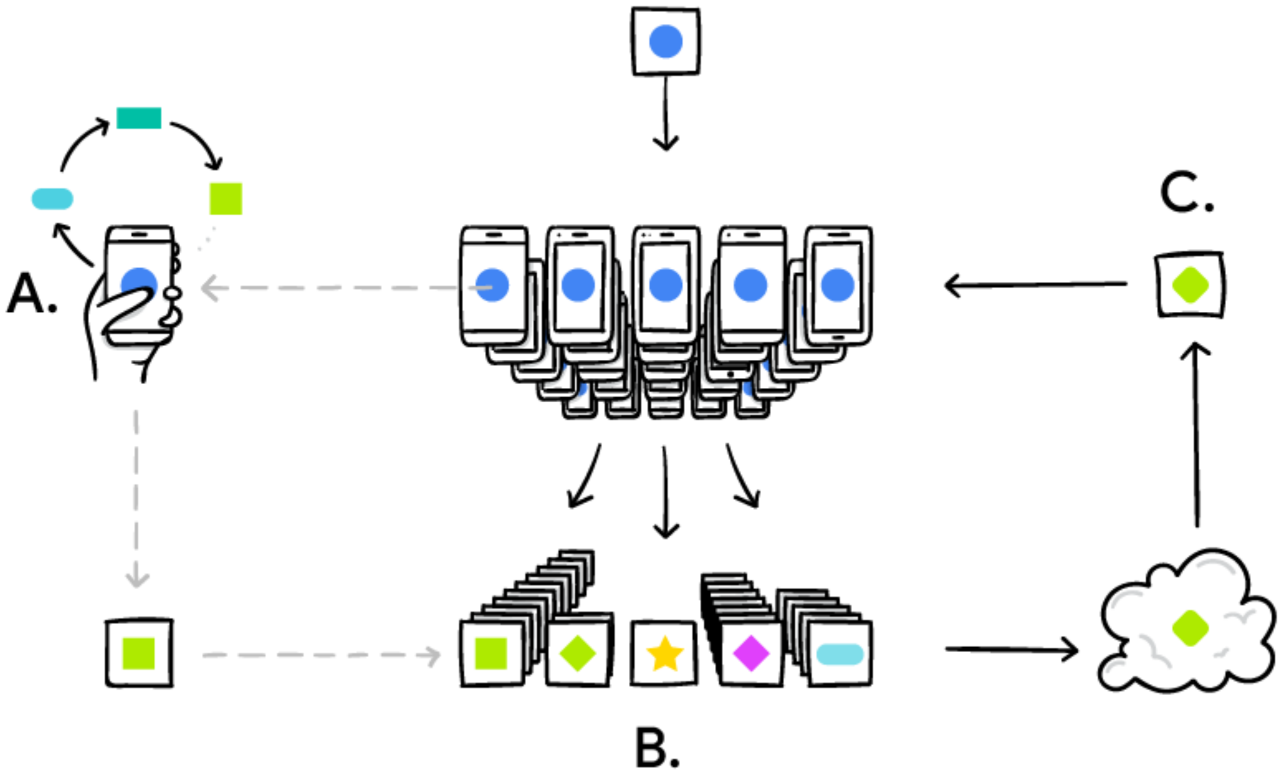


A. - phone personalizes the model locally, based on user's usage

Federated Learning



A. - phone personalizes the model locally, based on user’s usage

B. - many users' updates are aggregated

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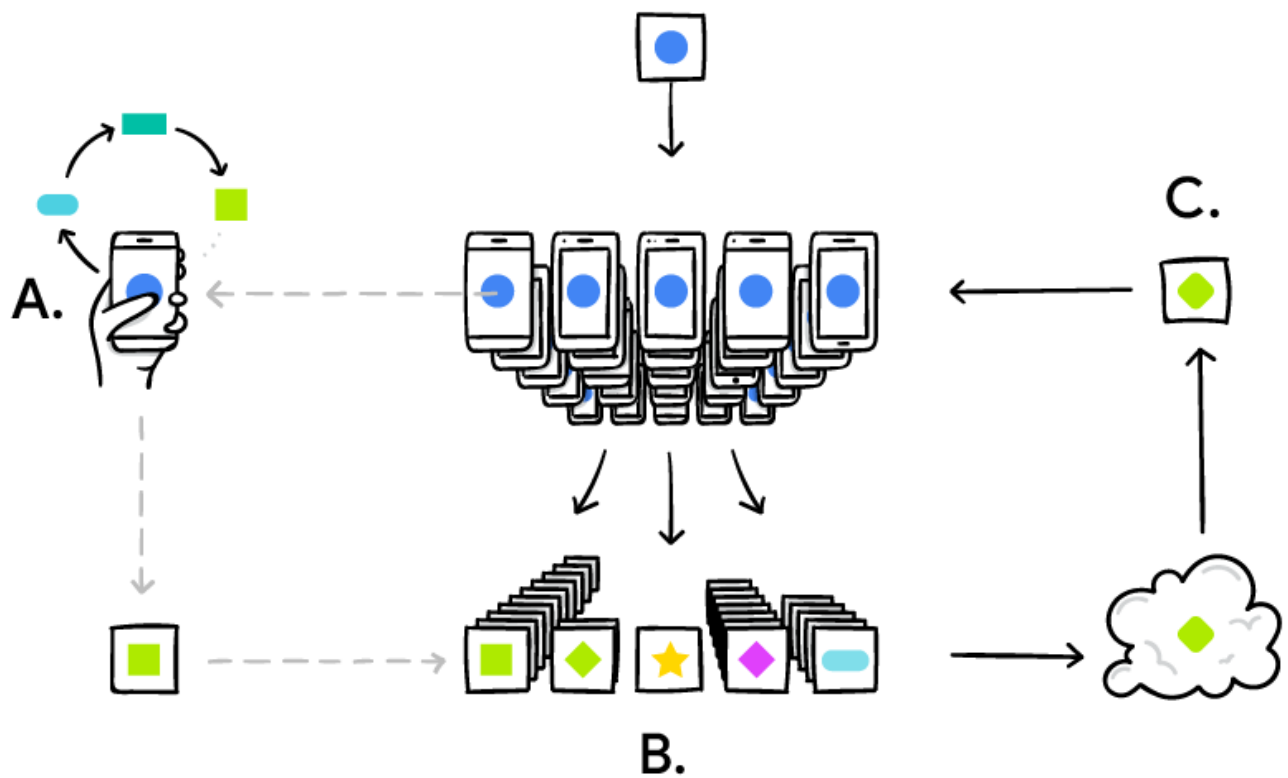

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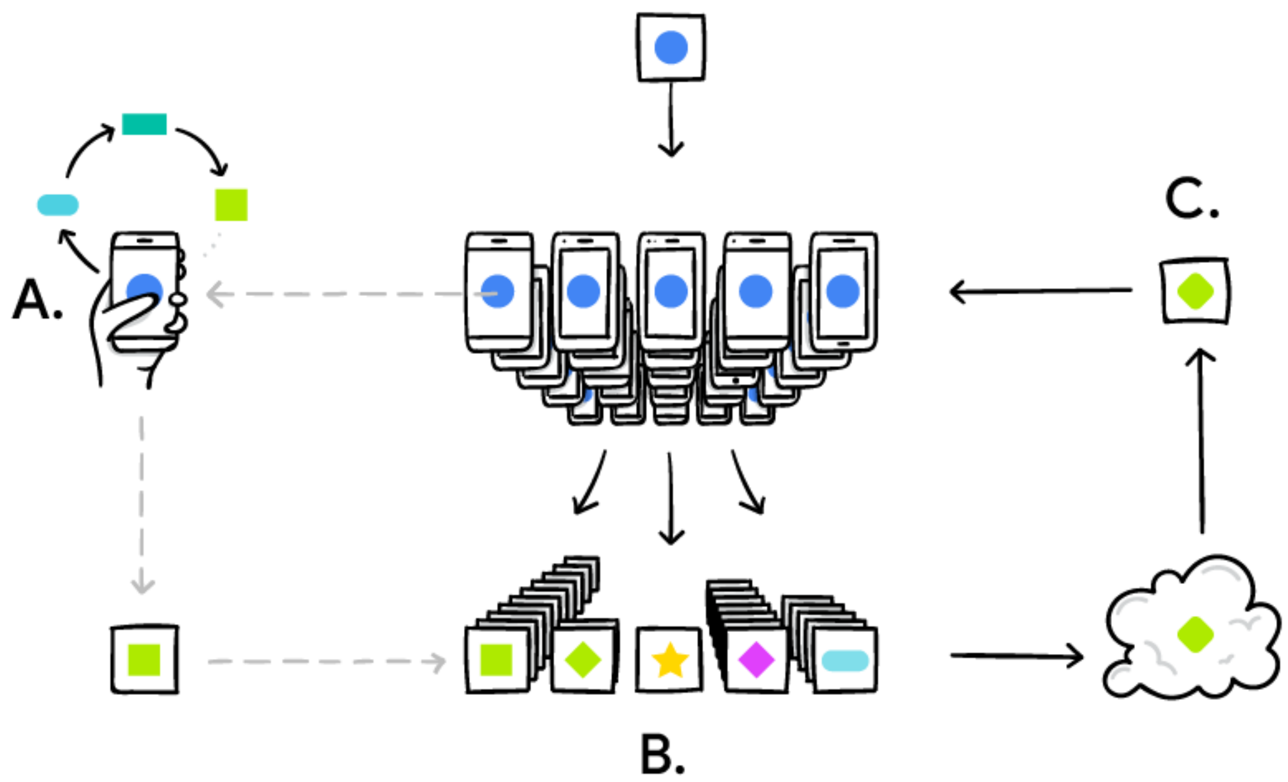

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As a result: a lot of updates, with cheap computations

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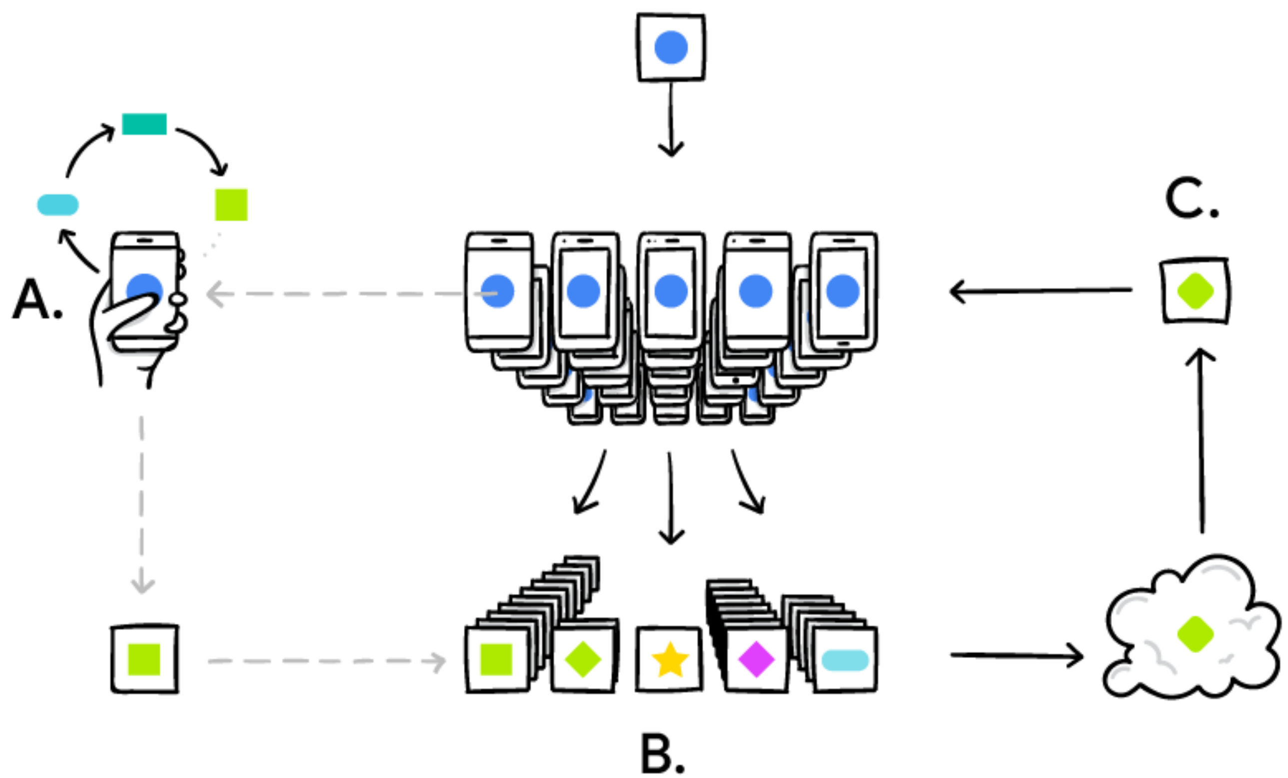

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Let's make communications cheaper!

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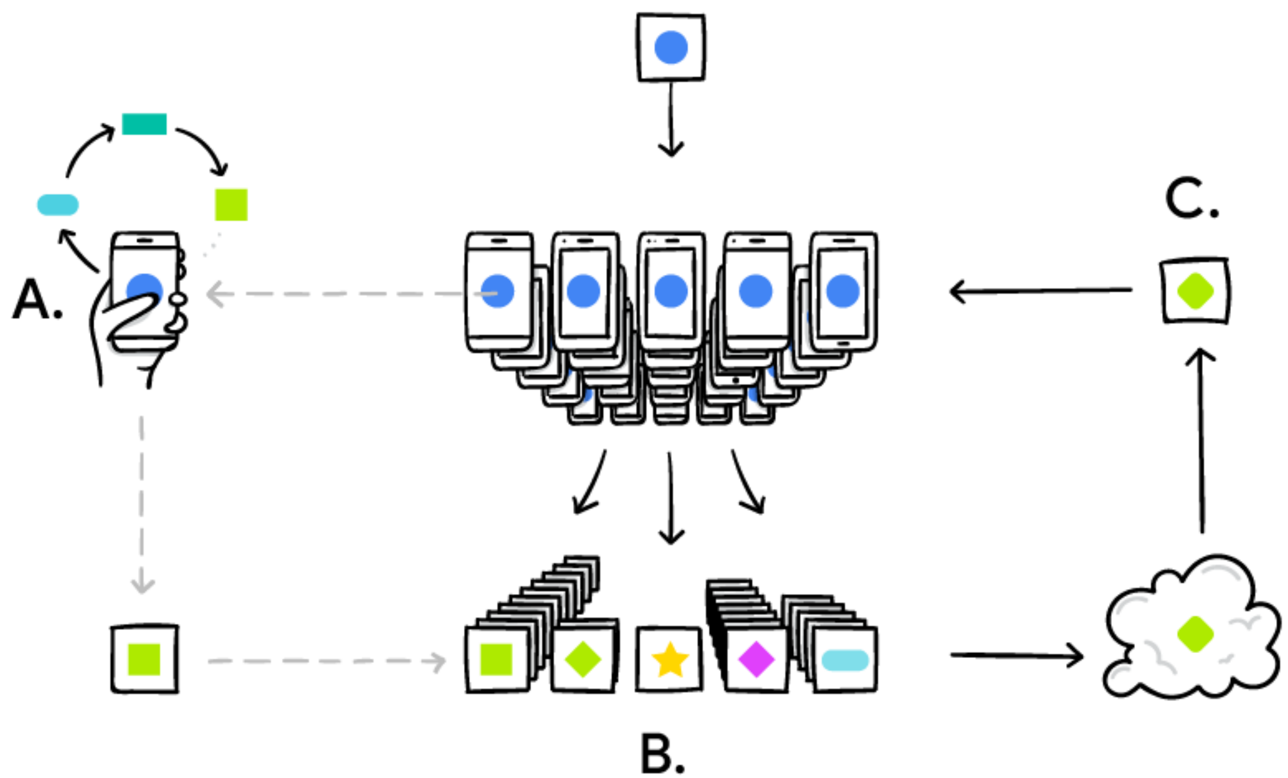

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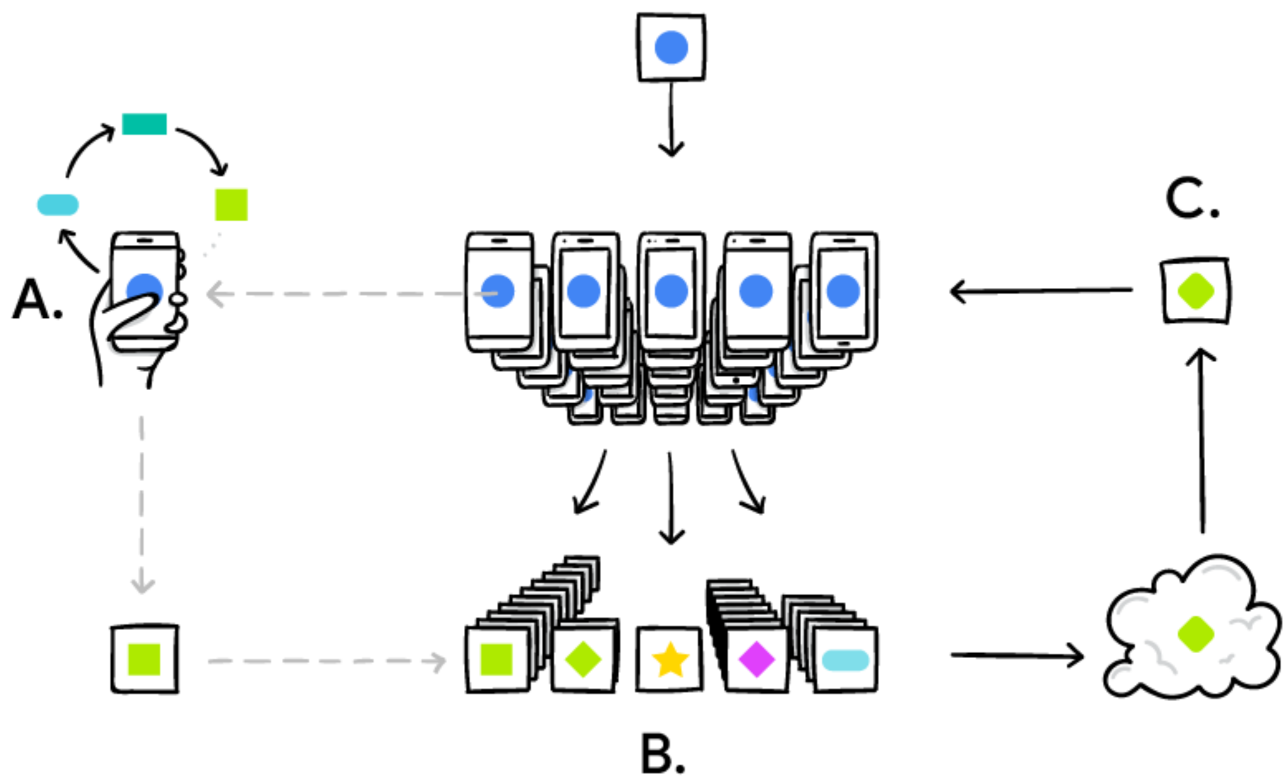

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Less often **Smaller size**

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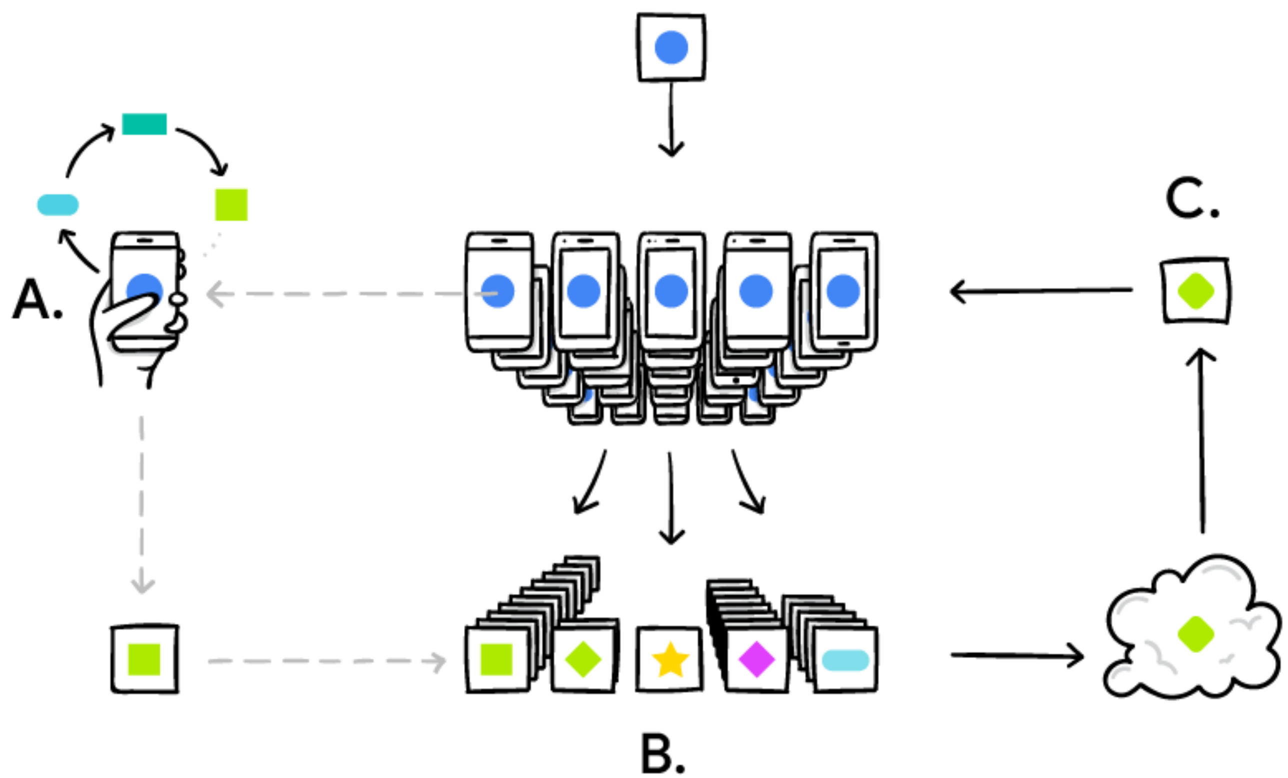

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Let's make communications cheaper!

Less often

Smaller size

The Size Is Important!

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Not so many fans

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Not so many fans → Everyone can go to the stadium and watch the game

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A lot of fans → Need to make a ballot for the tickets

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||

<u>**sparsification**</u>

Synchronous?

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Drawbacks of synchronous algorithms

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Wasting of time

All machines have to wait the
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A lot of communications in one time

Master machine communicates with
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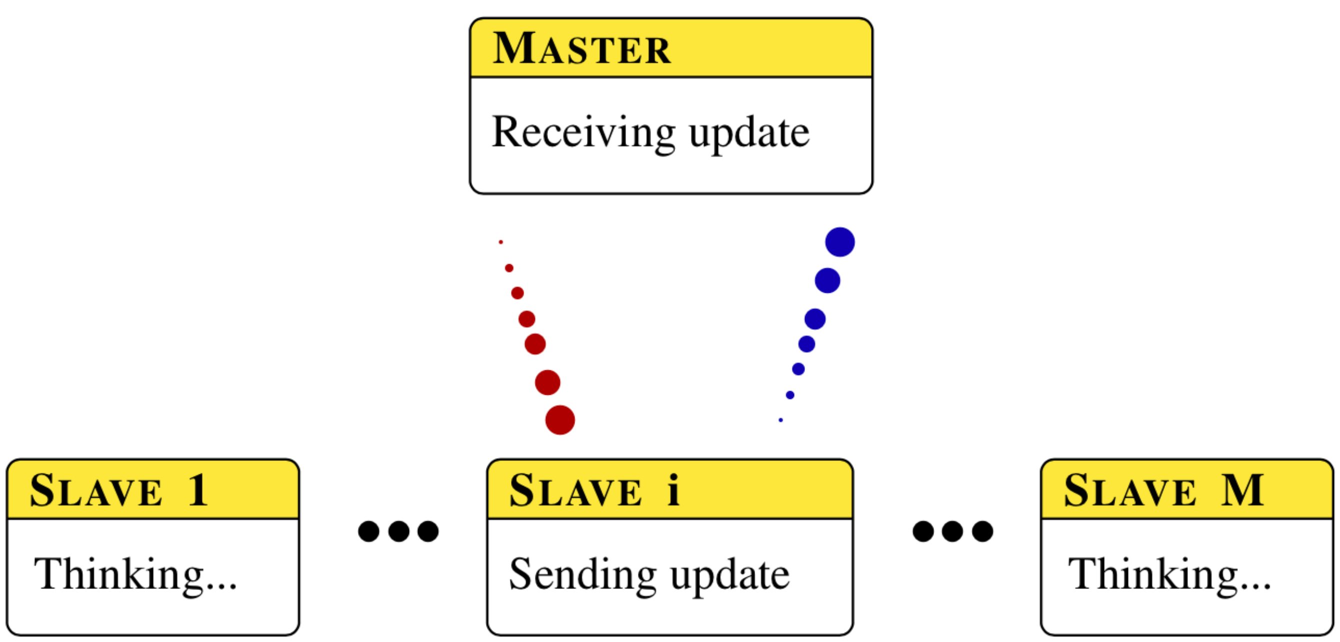
Let's kill 2 birds with one stone



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Each worker send its gradient and master calculate the weighted average

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$$F(x) = \sum_{i=1}^{M} \pi_i f_i(x) \quad \longrightarrow \quad \nabla F(x) = \sum_{i=1}^{M} \pi_i \nabla f_i(x)$$

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Asynchronous updates bring some delays

Gradient that master receive from worker is computed in one of previous iterate points

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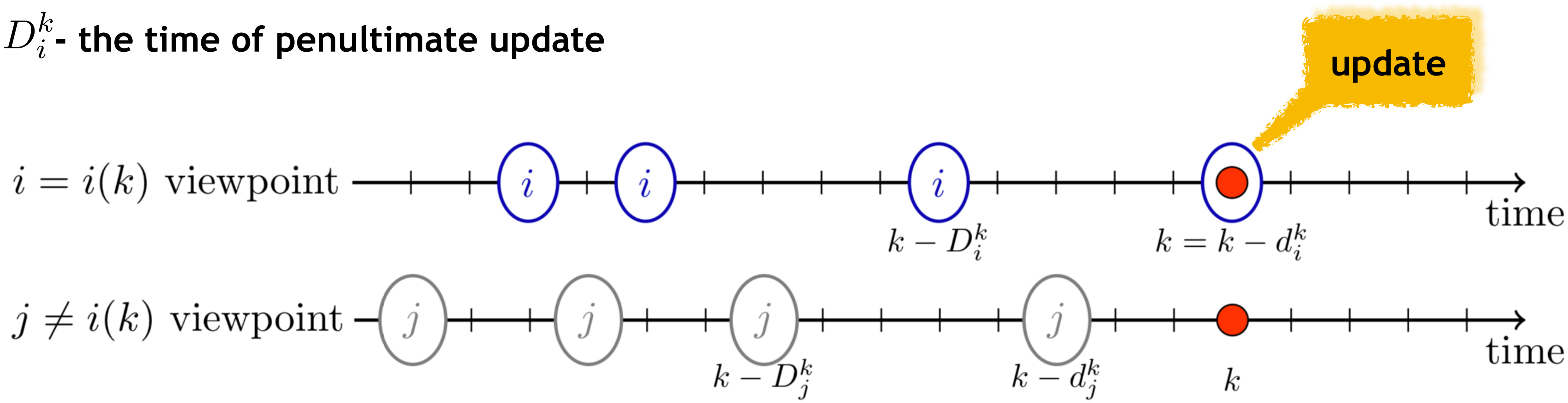

Non-sparsified Algorithm

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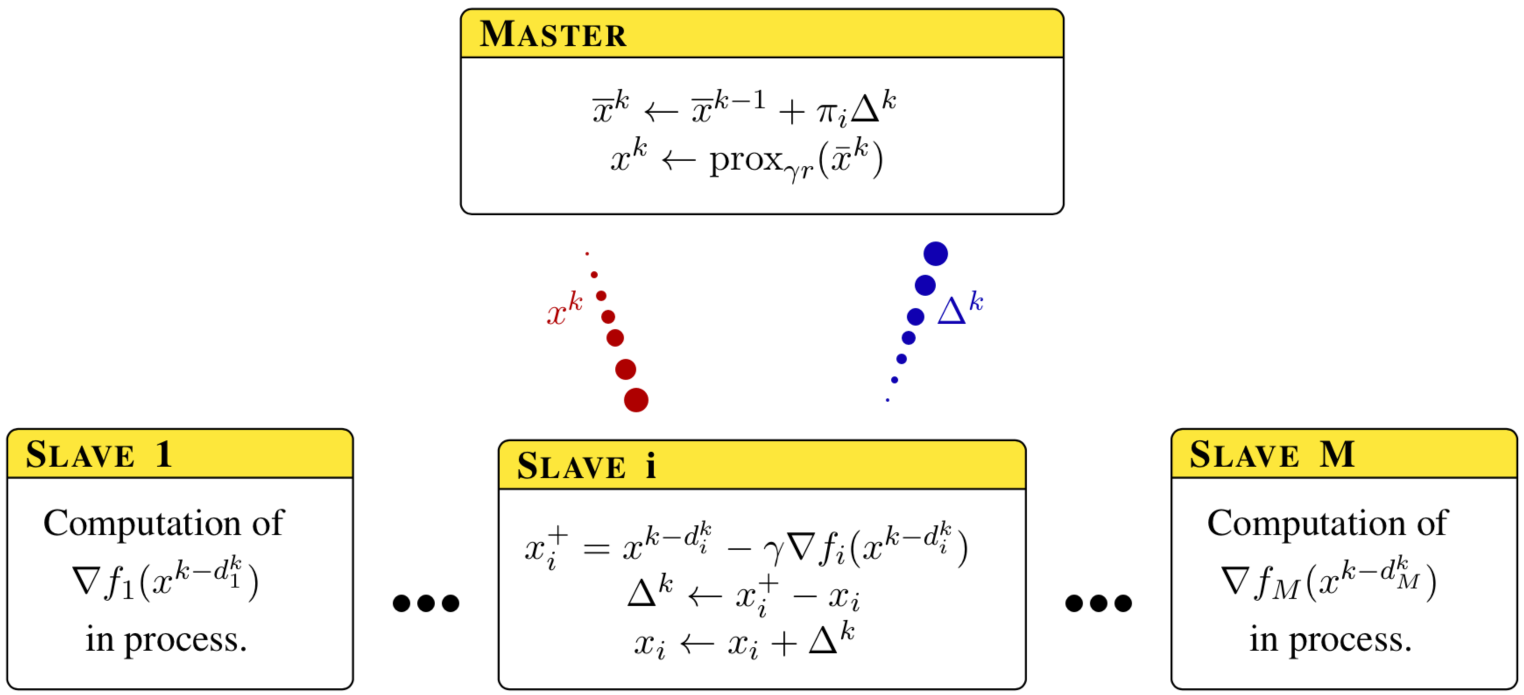

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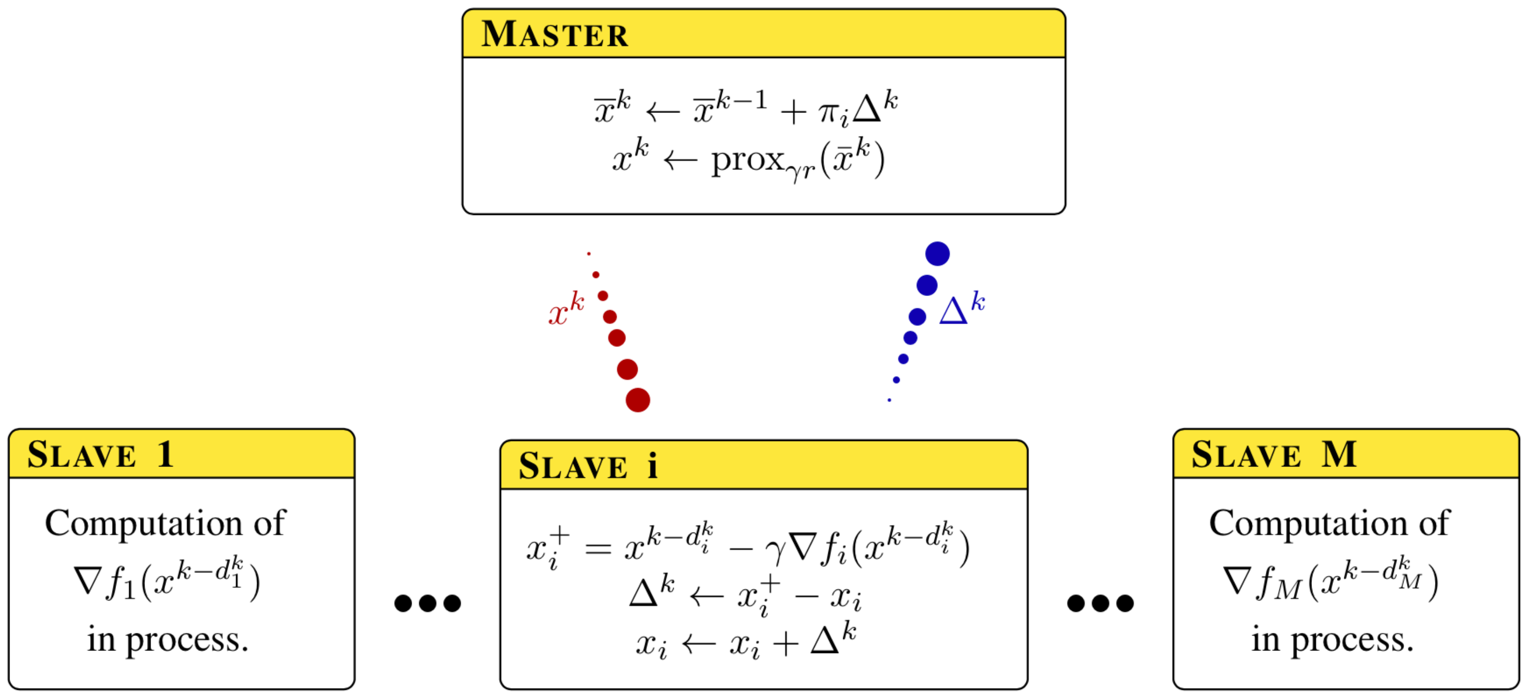

If dimension is very big, it is very expansive to send full gradient

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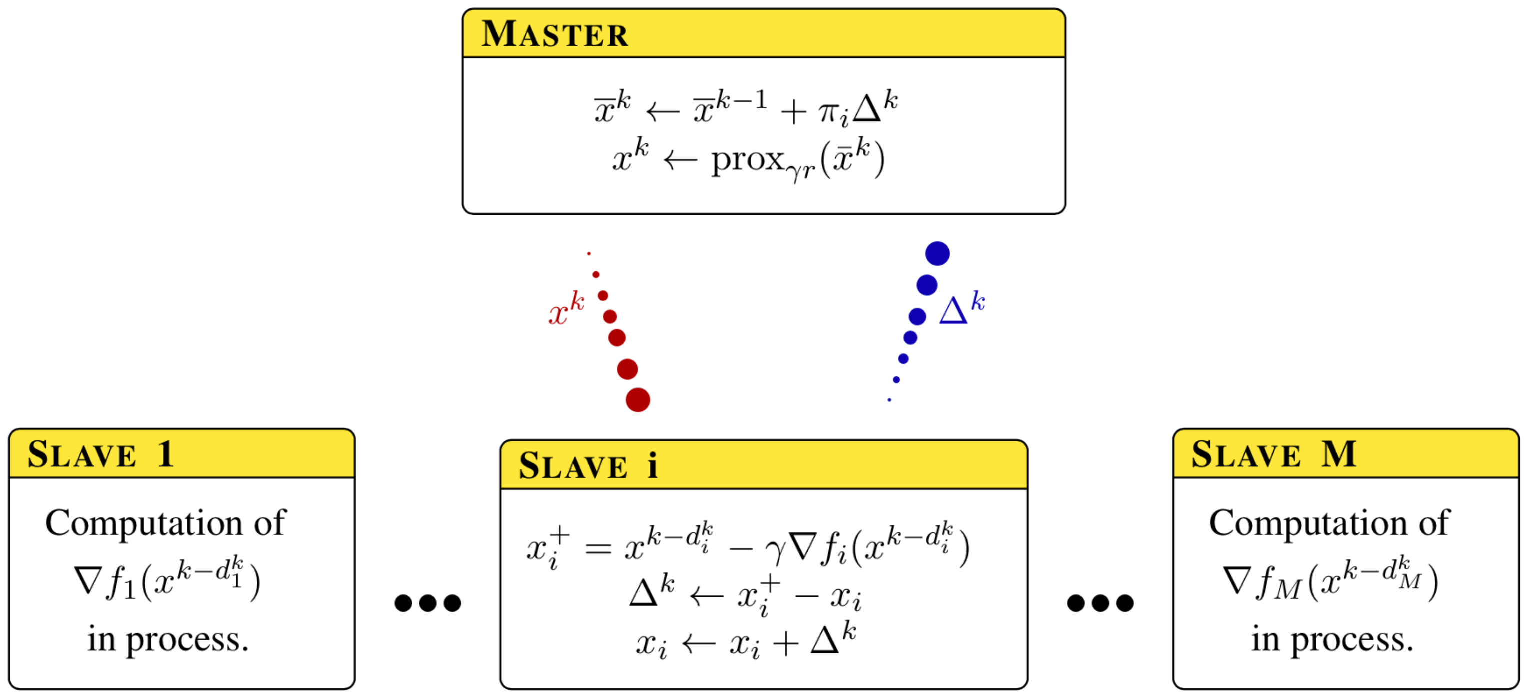

If dimension is very big, it is very expansive to send full gradient

If regulariser is sparsity enforcing, no need to sparsify master $\rightarrow$ worker

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A delay-tolerant proximal-gradient algorithm for distributed learning.
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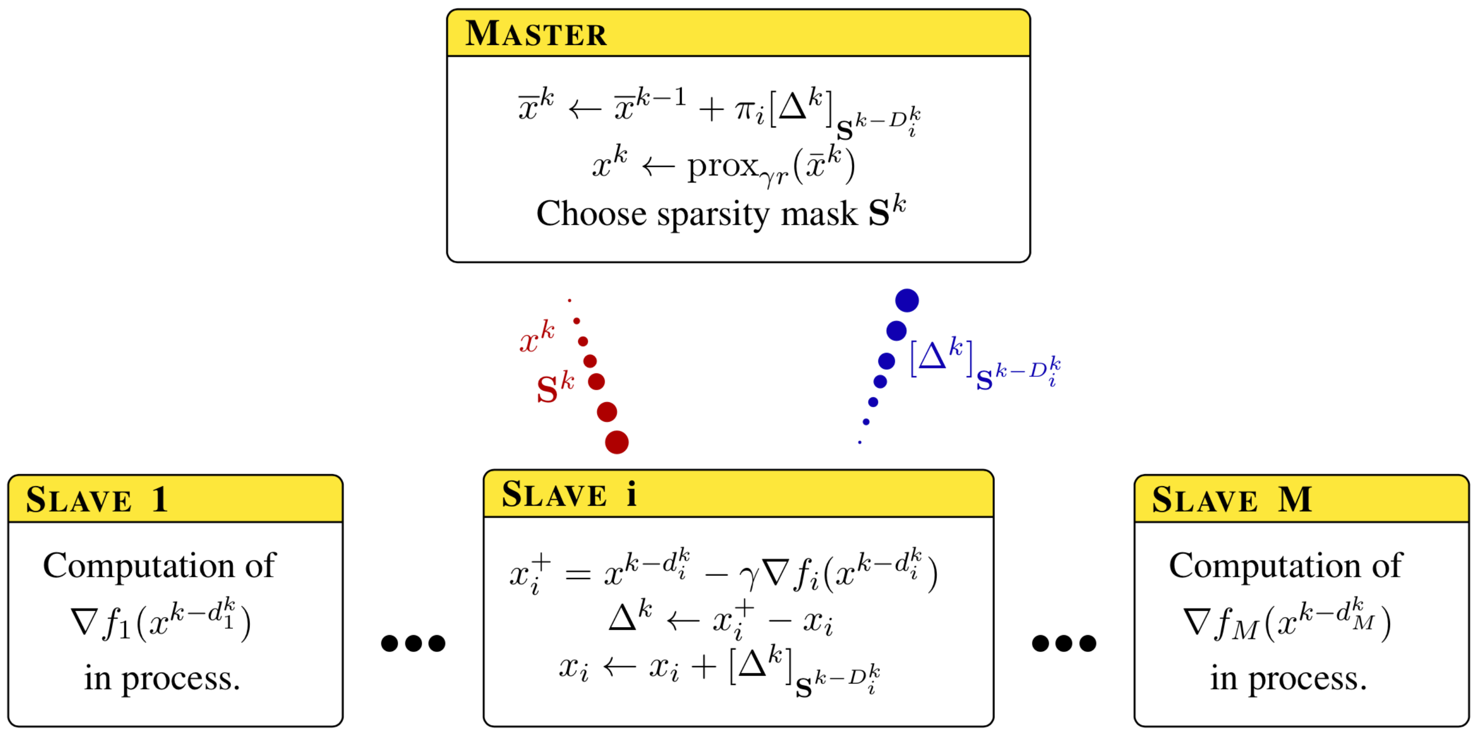
Fadili J., Malick J., Peyré G. Sensitivity analysis for mirror-stratifiable convex functions
SIAM Journal on Optimization. – 2018. – T. 28. – №. 4. – C. 2975-3000.

Sparsified Algorithm



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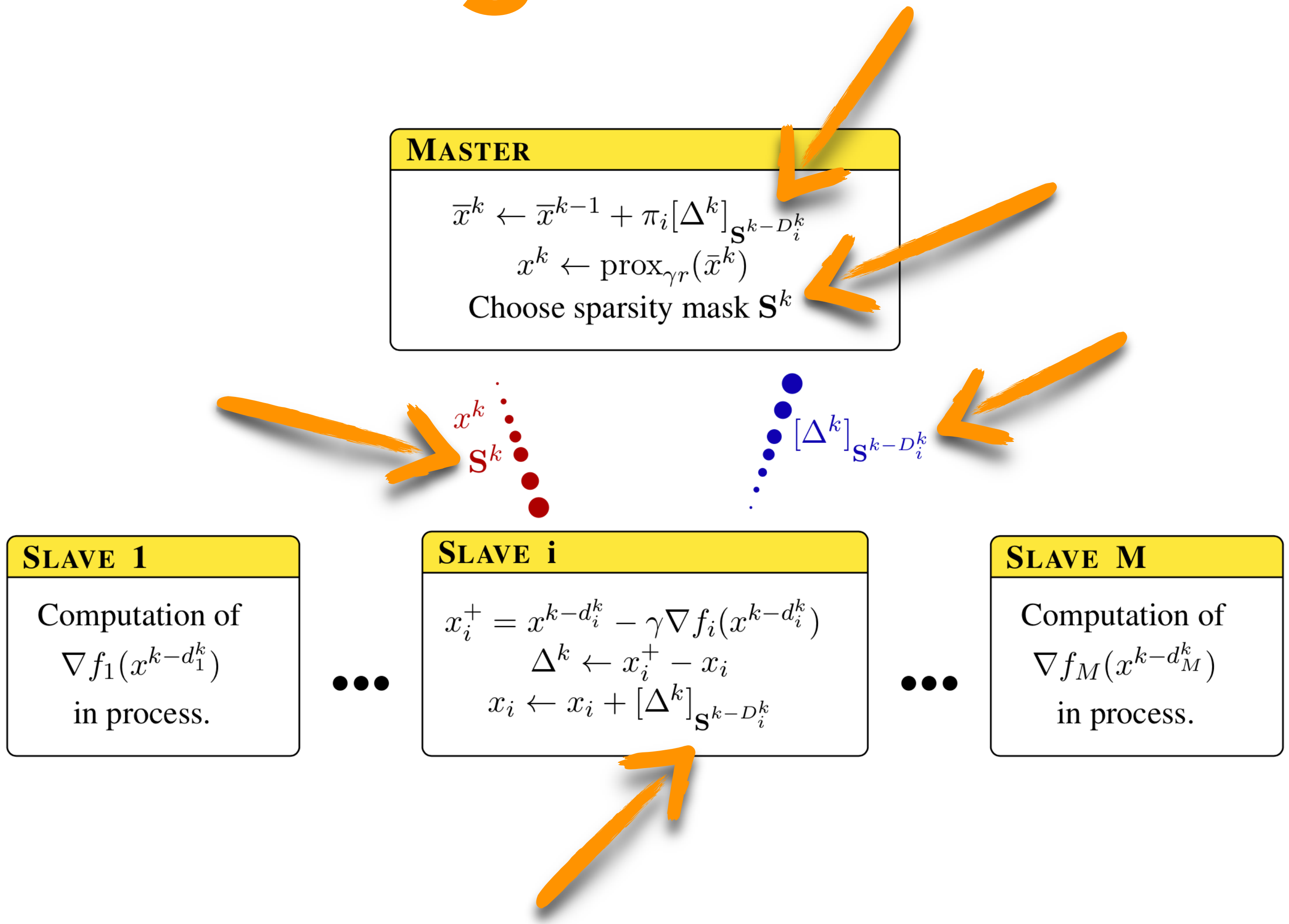

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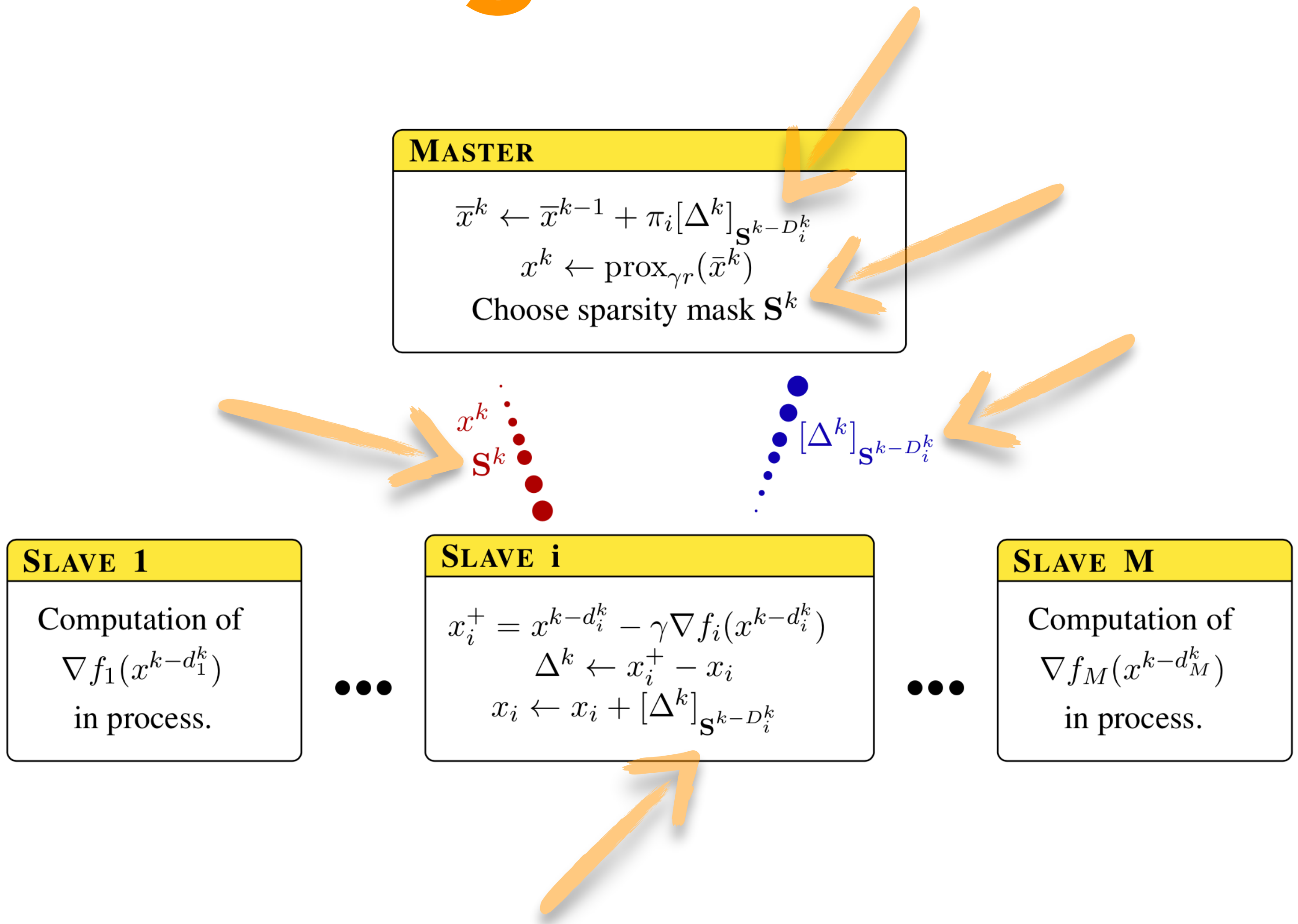

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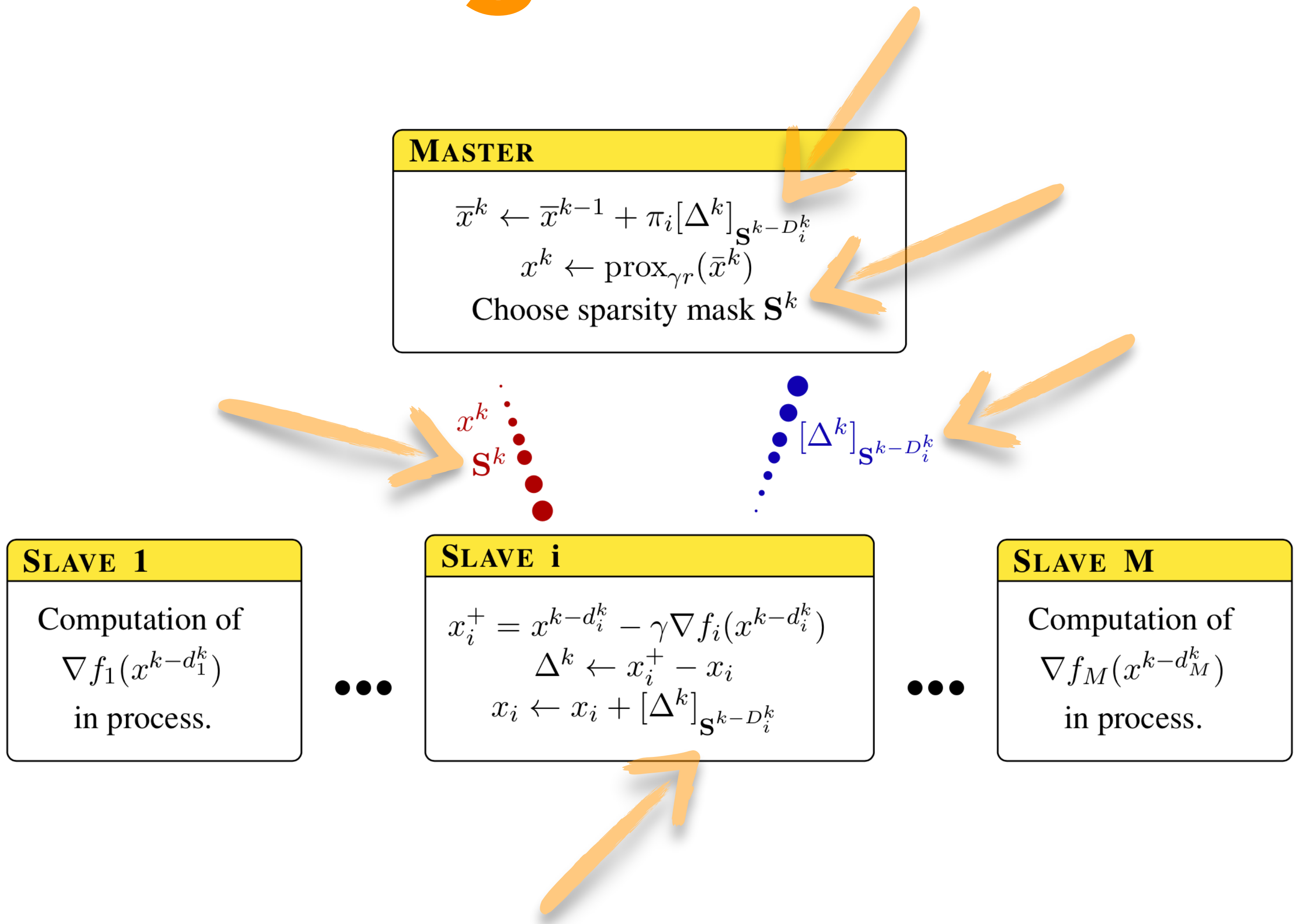
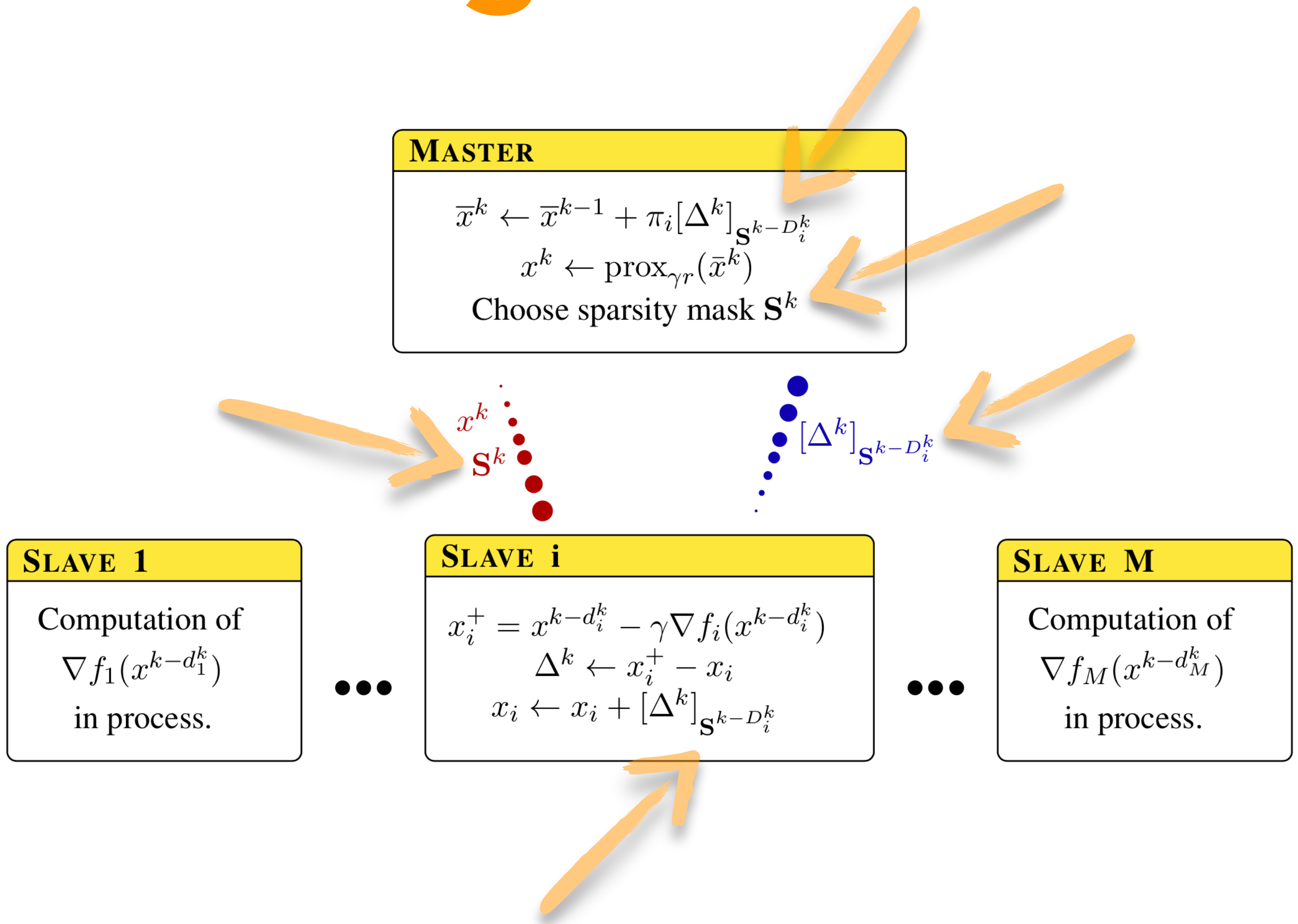


How to chose the sparsity mask?

Sparsified Algorithm



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Random uniform coordinate selection (Coordinate Descent with uniform probabilities)

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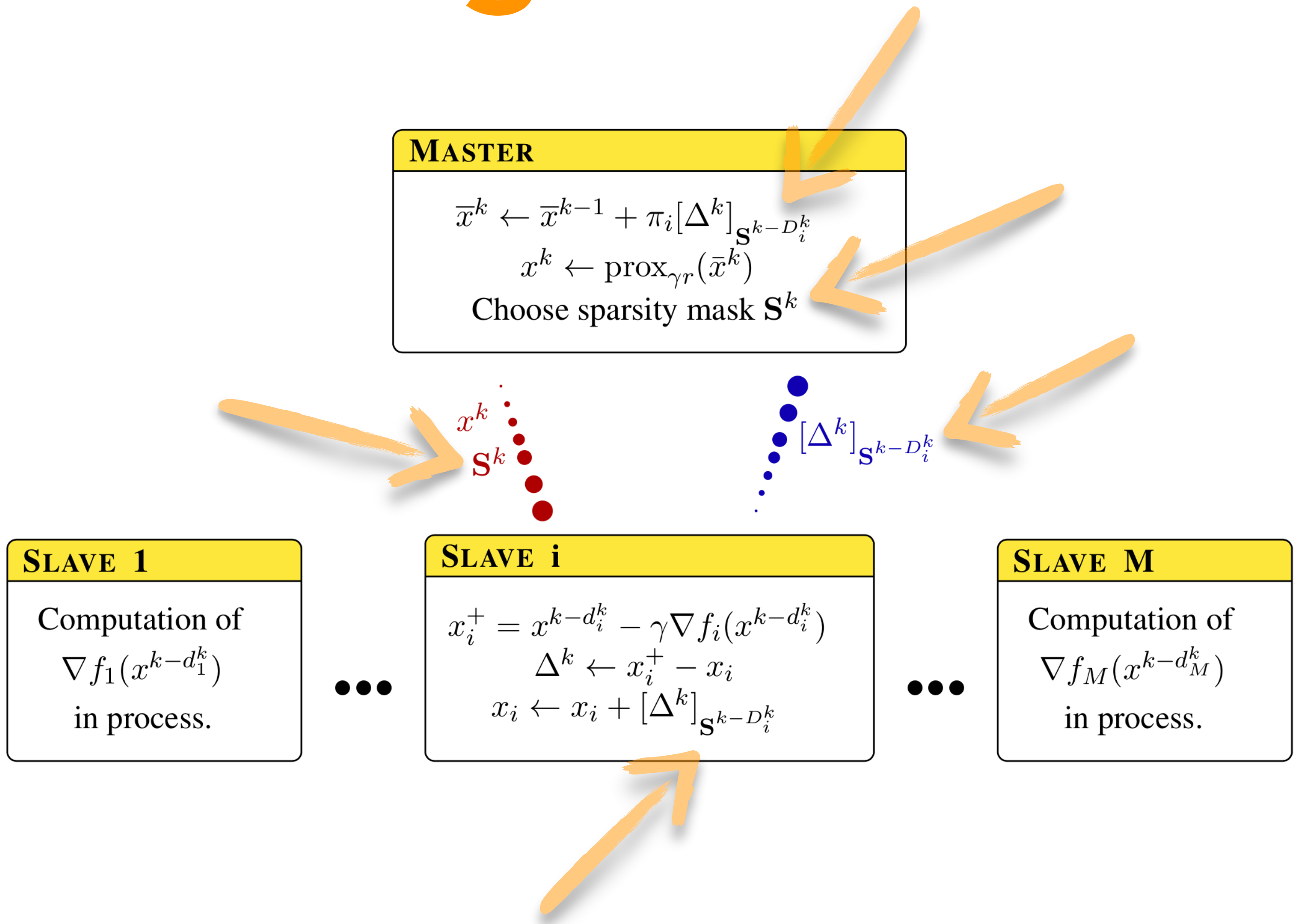

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Back to Ballot

It’s an example of identification from real world

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1 wave - most loyal fans (the biggest amount of games visited)

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Most loyal have probability 1, all the others much smaller

It’s an example of identification from real world

l_1-regularizer*

* - l_1-regularizer enforces coordinate sparsity

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Let's introduce our adaptive way of mask selection

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Theoretical result (for s.c. objective)

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Theoretical result (for s.c. objective)

$$\mathbb{E}\|x^k - x^\star\|^2 \leq \left(1 - 2\frac{\gamma p \mu L}{\mu + L}\right)^m \max_{i=1,..,M} \|x_i^0 - x_i^\star\|^2$$

where $k \in [k_m, k_{m+1})$ **and** $k_{m+1} = \min\left\{k : k - D_i^k \geq k_m \text{ for all } i\right\}$

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Numerical Experiments

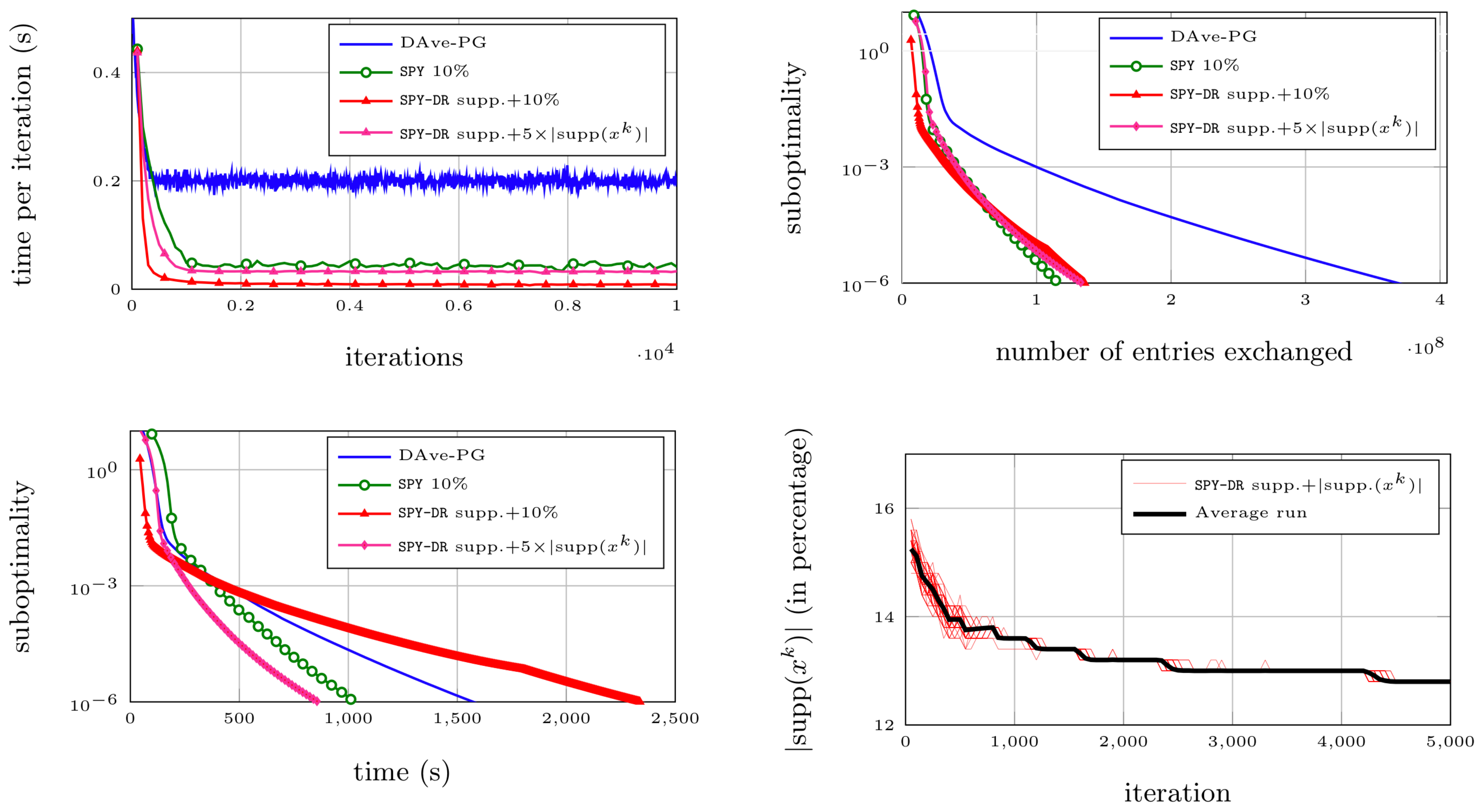

Figure 1: Logistic regression for the rcv1 dataset: evolution of the time per iteration, wallclock time performance, suboptimality vs communication, and robustness of identification.

Thank You For Your Attention